1.3 Answers

1.3.1 Solution to Example

The key idea is (as always) to solve this equation by doing the same to both sides, in this case integrating both sides with respect to *x*.

$$\frac{dy}{dx} = -x^3 + \frac{4x - 5}{2x^3}$$

$$\int \frac{dy}{dx}\,dx = \int -x^3 + \frac{4x - 5}{2x^3}\,dx$$

$$y = \int -x^3 + \frac{4x}{2x^3} - \frac{5}{2x^3}\,dx$$

$$= \int -x^3 + 2x^{-2} - \frac{5x^{-3}}{2}\,dx$$

$$= -\frac{x^4}{4} - 2x^{-1} + \frac{5x^{-2}}{4} + c$$

Given that $y = 7$ at $x = 1$,

$$7 = -\frac{1}{4} - 2 + \frac{5}{4} + c$$

$$\therefore\ c = 8$$

Thus, $$y = -\frac{x^4}{4} - 2x^{-1} + \frac{5x^{-2}}{4} + 8$$

[6 marks]

1.3.2 Solutions to Exercise

Answer 1

$$\frac{dy}{dx} = \cos x$$

Integrate both sides wrt x

$$\int \frac{dy}{dx}\,dx = \int \cos x\ dx$$

$$y = \sin x + c$$

Given that $y = 0.5$ at $x = \frac{\pi}{2}$,

$$0.5 = \sin\left(\frac{\pi}{2}\right) + c$$

$$0.5 = 1 + c$$

$$c = -0.5$$

Thus;

$$y = \sin x - 0.5$$

[4 marks]

Answer 2

$$\frac{dy}{dx} = sec^2 x$$

Integrate both sides with respect to x

$$\int \frac{dy}{dx}\, dx = \int sec^2 x \ dx$$

$$y = tan\, x + c$$

Given that $y = 1$ at $x = \frac{3\pi}{4}$,

$$1 = tan\left(\frac{3\pi}{4}\right) + c$$

$$1 = -1 + c$$

$$c = 2$$

Thus;

$$y = tan\, x + 2$$

[5 marks]

Answer 3
(i)

$$v = \frac{ds}{dt} = t^2 - t + 4$$

Integrate both sides with respect to t

$$\int \frac{ds}{dt}\, dt = \int t^2 - t + 4 \ dt$$

$$s = \frac{t^3}{3} - \frac{t^2}{2} + 4t + c$$

Given that $s = 100$ at $t = 6$,

$$100 = 72 - 18 + 24 + c$$

$$c = 22$$

Thus;

$$s = \frac{t^3}{3} - \frac{t^2}{2} + 4t + 22$$

[3 marks]

(ii)

When $t = 3$ s,

$$s = 9 - 4.5 + 12 + 22$$

$$s = 38.5 \text{ cm}$$

[2 marks]

Answer 4

$$\frac{dy}{dx} = x^2 \left(x^3 + 5 \right)^4$$

Integrate both sides with respect to x

$$\int \frac{dy}{dx}\, dx = \frac{1}{3} \int 3\,x^2 \left(x^3 + 5 \right)^4 \, dx$$

$$y = \frac{1}{3} \times \frac{\left(x^3 + 5 \right)^5}{5} + c$$

Given that $y = 8$ at $x = 0$,

$$209 = \frac{5^5}{15} + c$$

$$c = \frac{2}{3}$$

Thus;

$$y = \frac{\left(x^3 + 5 \right)^5}{15} + \frac{2}{3}$$

[5 marks]

Answer 5

(i)

$$5x \frac{dy}{dx} - 1 = 0$$

$$5x \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{5x}$$

Integrate both sides with respect to x

$$\int \frac{dy}{dx}\, dx = \frac{1}{5} \int \frac{1}{x}\, dx$$

$$y = \frac{1}{5}\, ln\left| x \right| + c$$

[4 marks]

(ii) Given that $y = 0$ at $x = 3$,

$$0 = \frac{1}{5}\, ln\, 3 + c$$

$$\therefore\ c = -\frac{1}{5}\, ln\, 3$$

Thus,

$$y = \frac{1}{5}\, ln\left| x \right| - \frac{1}{5}\, ln\, 3$$

$$y = \frac{1}{5}\, ln \left| \frac{x}{3} \right|$$

[3 marks]

Answer 6

(i)

$$sec\,x\,\frac{dy}{dx} - x = 0$$

$$sec\,x\,\frac{dy}{dx} = x$$

$$\frac{dy}{dx} = \frac{x}{sec\,x}$$

$$\frac{dy}{dx} = x\,cos\,x$$

Integrate both sides wrt x

$$\int \frac{dy}{dx}\,dx = \int x\,cos\,x\;\;dx$$

$$\textbf{\textit{L}}\quad \textbf{\textit{I}}\qquad\quad \textbf{\textit{D}}\qquad \textbf{\textit{I}}$$

$$y = x\,sin\,x - \int sin\,x\;\;dx$$

$$= x\,sin\,x + cos\,x + c$$

[4 marks]

(ii)

$$\pi = \frac{\pi}{2}\,sin\left(\frac{\pi}{2}\right) + cos\left(\frac{\pi}{2}\right) + c$$

$$\pi = \frac{\pi}{2} + c$$

$$\therefore\; c = \frac{\pi}{2}$$

Thus;

$$y = x\,sin\,x + cos\,x + \frac{\pi}{2}$$

[3 marks]

Answer 7

$$\text{If } x = 3 \tan u \text{ then } \frac{dx}{du} = 3 \sec^2 u$$

$$\frac{dy}{dx} = \frac{1}{9 + x^2}$$

Integrate both sides wrt x

$$\int \frac{dy}{dx}\, dx = \int \frac{1}{9 + (3 \tan u)^2} \; 3 \sec^2 u \; du$$

$$y = \int \frac{1}{9(1 + \tan^2 u)} \; 3 \sec^2 u \; du$$

$$y = \frac{1}{3} \int \frac{1}{\sec^2 u} \sec^2 u \; du$$

$$y = \frac{1}{3} \int 1 \, du$$

$$y = \frac{1}{3} u + c$$

Given that $y = \frac{\pi}{9}$ at $x = \sqrt{3}$ when $u = \frac{\pi}{6}$,

$$\frac{\pi}{9} = \frac{\pi}{18} + c$$

$$c = \frac{\pi}{18}$$

Thus;

$$y = \frac{1}{3} \arctan\left(\frac{x}{3}\right) + \frac{\pi}{18}$$

[8 marks]

Answer 8

(i)

$$t \frac{ds}{dt} = t^2 + 4$$

$$\frac{ds}{dt} = t + \frac{4}{t}$$

Integrate both sides wrt t

$$\int \frac{ds}{dt}\, dt = \int t + \frac{4}{t} \; dt$$

$$s = \frac{t^2}{2} + 4\, ln|\, t\,| + c$$

[5 marks]

(ii) Using the given boundary that $s = 4\, ln\, 2$ when $t = 2$, yields;

$$4\, ln\, 2 = \frac{2^2}{2} + 4\, ln\, 2 + c$$

$$c = -2$$

$$s = \frac{t^2}{2} + 4\, ln|\, t\,| - 2$$

So, when $t = 4$

$$s = \frac{t^2}{2} + 4\, ln|\, t\,| - 2$$

$$= \frac{4^2}{2} + 4\, ln\, 4 - 2$$

$$= 8 + 4\, ln\, 2^2 - 2$$

$$= 6 + 8\, ln\, 2$$

That is,

$$a = 6 \qquad \text{and} \qquad b = 8$$

[4 marks]

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