

Lesson 8

A-Level Pure Mathematics : Year 2 Differential Equations I

8.1 Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 40

Question 1

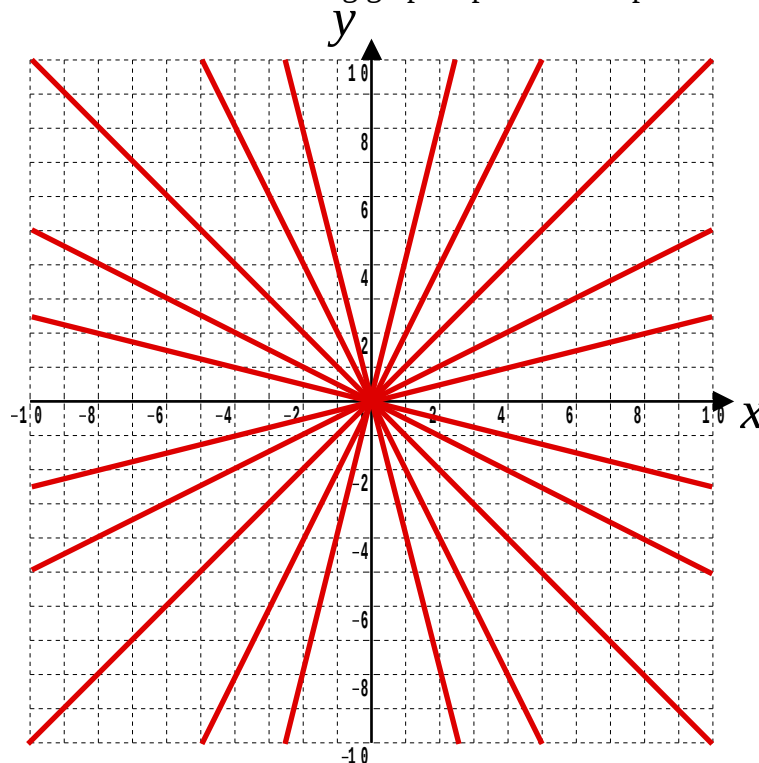
- (i) Determine the general solution to the differential equation, $\frac{dy}{dx} = \frac{y}{x}$

[3 marks]

- (ii) The general solution is represented by all possible straight lines through the origin. Given that $y = 8$ when $x = 4$ find the particular solution.
Present your solution as elegantly as possible and in the form $y = f(x)$.

[1 mark]

- (iii) Which line on the following graph represents the particular solution ?



[1 mark]

Question 2

A-Level Examination Question from June 2018, Paper 1, Q10 (Edexcel)

The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation,

$$\frac{dH}{dt} = \frac{H \cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) Show that $H = 5e^{0.1 \sin(0.25t)}$

[5 marks]

(b) State the maximum height of the passenger above the ground.

[1 mark]

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

(c) Find the value of T

[2 marks]

Question 3

Show that the differential equation,

$$\frac{dy}{dx} = x e^{2y} \quad \text{where} \quad y = 0 \quad \text{when} \quad x = 0$$

has solution,

$$y = \ln\left(\frac{1}{\sqrt{1-x^2}}\right) \quad |x| < 1$$

[4 marks]

Question 4

A-Level Examination Question from June 2013, Paper C4, Q6 (Edexcel)

Water is being heated in a kettle.

At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda (120 - \theta), \quad \theta \leq 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$

(a) solve this differential equation to show that

$$\theta = 120 - 100 e^{-\lambda t}$$

[8 marks]

When the temperature of the water reaches 100°C , the kettle switches off.

- (b)** Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

[3 marks]

Question 5

A-Level Examination Question from January 2007, Paper C4, Q4 (Edexcel)

(a) Express in partial fractions;

$$\frac{2x - 1}{(x - 1)(2x - 3)}$$

[3 marks]

(b) Given that $x \geq 2$, find the general solution of the differential equation

$$(2x - 3)(x - 1) \frac{dy}{dx} = (2x - 1)y$$

[5 marks]

- (c) Hence find the particular solution of this differential equation that satisfies $y = 10$ at $x = 2$, giving your answer in the form $y = f(x)$

[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com