A-Level Pure Mathematics
Year 1, Year 2 Revision
Grade Grabber 1 Solutions

Answer 1

$$\sum_{m=1}^{k} (7m + 3) = 777$$

$$10 + 17 + 24 + \dots + (7k + 3) = 777$$

The sequence is is Arithmetic Progression; $a = 10, \quad d = 7, \quad n = k$

$$Sum = \frac{n}{2}\{2a + (n - 1)d\}$$

$$\frac{k}{2}\{20 + (k - 1)7\} = 777$$

$$20k + 7k^2 - 7k = 1554$$

$$7k^2 + 13k - 1554 = 0$$

$$k = \frac{-13 \pm \sqrt{13^2 - 4 \times 7 \times (-1554)}}{2 \times 7}$$

$$= 14 \quad \text{or} \quad -\frac{111}{7}$$

$$\therefore k = 14$$

[4 marks]

Answer 2

Let $x = -1$: Then $x^2 = 1$ and $(x + 1)^2 = 0$

This makes the given statement False ☐

In fact, any $x < -0.5$ makes the given statement false

[2 marks]

Answer 3

$$5x^2 + k = 3x + 7$$

$$5x^2 - 3x + k - 7 = 0$$

To have two distinct real roots the Discriminant must be positive

$$D > 0$$

$$b^2 - 4ac > 0$$

$$(-3)^2 - 4 \times 5 \times (k - 7) > 0$$

$$9 - 20k + 140 > 0$$

$$20k < 149$$

$$k < 7.45$$

[4 marks]

Answer 4

$$Arc\ Length \ : \quad r\theta = 4$$

$$Sector\ Area \ : \ \frac{1}{2} r^2 \,(2\pi - \theta) = 12$$

$$\text{Combining by substitution yields} \ : \ \frac{1}{2} r^2 \left(2\pi - \frac{4}{r} \right) = 12$$

$$\pi r^2 - 2r - 12 = 0$$

$$r = \frac{2 \pm \sqrt{2^2 - 4\pi(-12)}}{2\pi}$$

$$r = 2.298 \quad \text{or} \ -1.662$$

$$\therefore \ r = 2.298 \text{ cm}$$

[6 marks]

Answer 5

(i) The binomial theorem states;

$$(1 + x)^n = 1 + nx + \frac{n (n - 1)}{2\,!} x^2 + \frac{n (n - 1) (n - 2)}{3\,!} x^3 + \ldots$$

... with $n = -3$ and x replaced with $(-x)$ yields...

$$(1 - x)^{-3} = 1 + (-3)(-x) + \frac{(-3)(-4)}{2} (-x)^2 + \frac{(-3)(-4)(-5)}{6} (-x)^3$$

$$= 1 + 3x + 6x^2 + 10x^3 + \ldots \qquad \text{valid for } |x| < 1$$

[4 marks]

(ii) The coefficients are the triangular numbers....

$$(1 - x)^{-3}$$

$$= 1 + 3x + 6x^2 + 10x^3 + 15x^4 + 21x^5 + 28x^6 + 36x^7 + 45x^8 + \ldots$$

$$\therefore \text{Coefficient of } x^8 \text{ is } 45$$

[2 marks]

Answer 6

$$y = e^{5x} - 1$$

$$x = e^{5y} - 1 \qquad \text{swapping input and output}$$

$$x + 1 = e^{5y}$$

$$ln(x + 1) = 5y$$

$$y = \frac{1}{5} ln(x + 1)$$

$$f^{-1}(x) = \frac{1}{5} ln(x + 1) \qquad x \in \mathbb{R},\ x > -1 \ \Leftarrow \ \text{Domian}$$

[4 marks]

Answer 7

$$M = \left(\frac{-4 + 12}{2}, \frac{0 + 12}{2} \right)$$

$$= (4. 6)$$

$$m_{AB} = \frac{12 - 0}{12 - (-4)}$$

$$= \frac{3}{4}$$

$$\therefore m_{pPERP} = -\frac{4}{3}$$

AB will have a perpendicular bisector with an equation of the form $y = mx + c$

$$6 = -\frac{4}{3} \times 4 + c$$

$$c = \frac{34}{3}$$

$$y = -\frac{4}{3}x + \frac{34}{3}$$

Strategy

The strategy is now to repeat the above process.
Pick two more points on the circumference of the circle, then get the perpendicular bisector of the line segment between those two points.

The centre of the circle is then where the two different perpendicular bisectors intersect.

Incidently, this is how you would physically do the question with a straight edge pencil and compass !

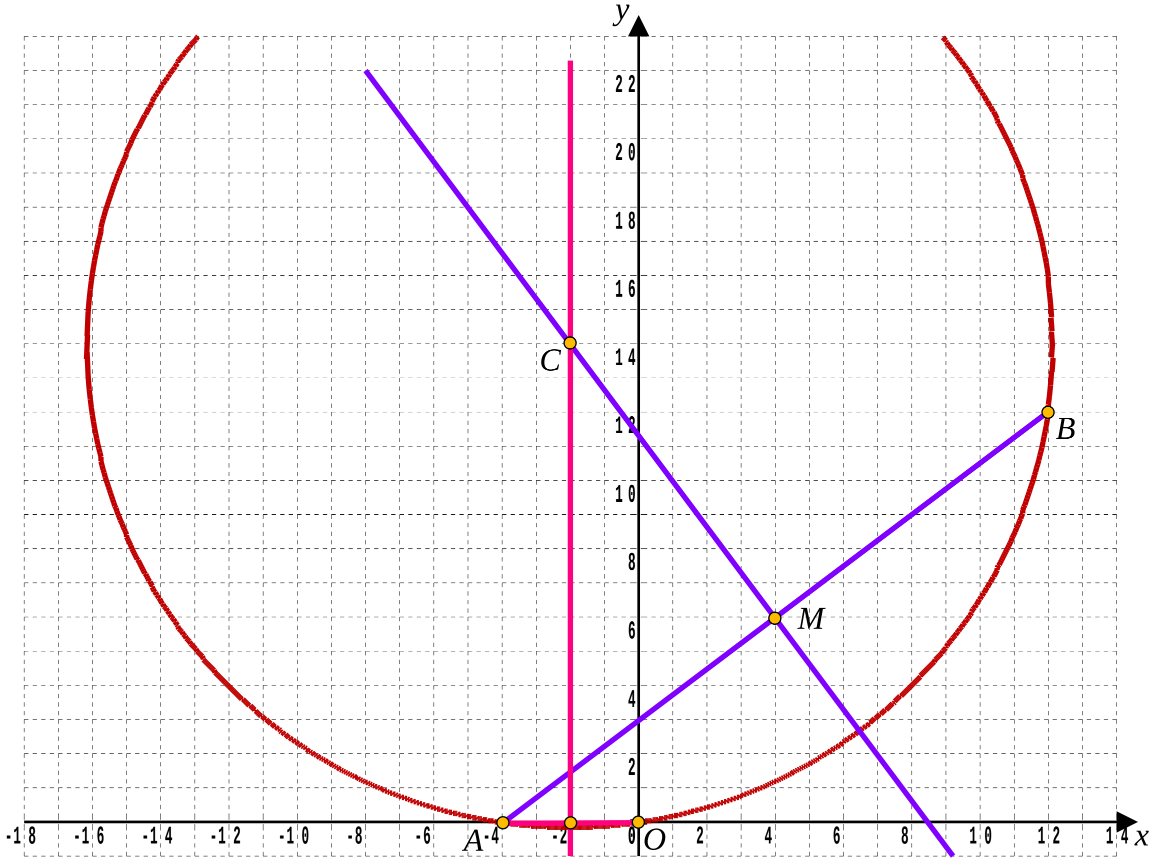
There is a clever way of getting the x-coordinate of the circle's centre using the fact that the circumference passes through both $A(-4, 0)$ and $O(0, 0)$ both on the x-axis, so the perpendicular bisector of the line segment between those two points will be the vertical line with equation $x = -2$.
As the circle centre must be somewhere on that perpendicular bisector the circle centre has $x = -2$

To find where $x = -2$ and $y = -\frac{4}{3}x + \frac{34}{3}$ intersect,

$$y = -\frac{4}{3}(-2) + \frac{34}{3}$$

$$y = 14$$

Here is a diagram of the situation,



Use Pythagoras' Theorem to get the radius: $\sqrt{(-2)^2 + 14^2} = \sqrt{200}$

The equation of the circle is thus: $(x + 2)^2 + (y - 14)^2 = 200$

[8 marks]