A-Level Pure Mathematics
Year 1, Year 2 Revision
Grade Grabber 2 Solutions

Answer 1

For a geometric progression

$$
\begin{aligned}
Sum_{\infty} &= \frac{a}{1 - r} \\
&= \frac{\sin 2\theta}{1 - \cos 2\theta} \\
&= \frac{2 \sin \theta \cos \theta}{\cos^2 \theta + \sin^2 \theta - (\cos^2 \theta - \sin^2 \theta)} \\
&= \frac{2 \sin \theta \cos \theta}{2 \sin^2 \theta} \\
&= \frac{\cos \theta}{\sin \theta} \\
&= \cot \theta \qquad \square
\end{aligned}
$$

[4 marks]

Answer 2

(a)

$$13\,sin\,\theta \;+\; 8\,cos\,\theta \;=\; 11$$

$$13\theta \;+\; 8\left(1 \;-\; \frac{\theta^2}{2}\right) \;=\; 11 \qquad \text{using small angle aooroximations}$$

$$13\theta \;+\; 8 \;-\; 4\,\theta^2 \;=\; 11$$

$$4\,\theta^2 \;-\; 13\theta \;+\; 3 \;=\; 0 \qquad \square$$

$$(\,4\theta \;-\; 1\,)(\,\theta \;-\; 3\,) \;=\; 0$$

$$\therefore\;\; \theta \;\in\; \{\,0.25,\;\; 3\}$$

The angle of 3 radians is too large to be a small angle approximation and so is rejected leaving $\theta = 0.25$ radians as the only solution.

[4 marks]

(b)

$$R\,sin\;(\,\theta \;+\; \alpha\,) \;=\; R\,sin\,\theta\;cos\,\alpha \;+\; R\,cos\,\theta\;sin\,\alpha$$

Comparison with $13\,sin\,\theta \;+\; 8\,cos\,\theta$ yields;

$$\frac{R\,sin\,\alpha}{R\,cos\,\alpha} \;=\; \frac{8}{13}$$

$$\alpha \;=\; arctan\left(\frac{8}{13}\right)$$

$$\alpha \;=\; 0.5517^c$$

$$R \;=\; \sqrt{8^2 \;+\; 13^2}$$

$$=\; 15.264$$

Thus,

$$13\,sin\,\theta \;+\; 8\,cos\,\theta \;=\; 11$$

$$15.264\,sin\;(\,\theta \;+\; 0.5517\,) \;=\; 11$$

$$sin\;(\,\theta \;+\; 0.5517\,) \;=\; 0.7206$$

$$\theta \;+\; 0.5517 \;=\; arcsin\,0.7206$$

$$\theta \;+\; 0.5517 \;=\; 0.8047$$

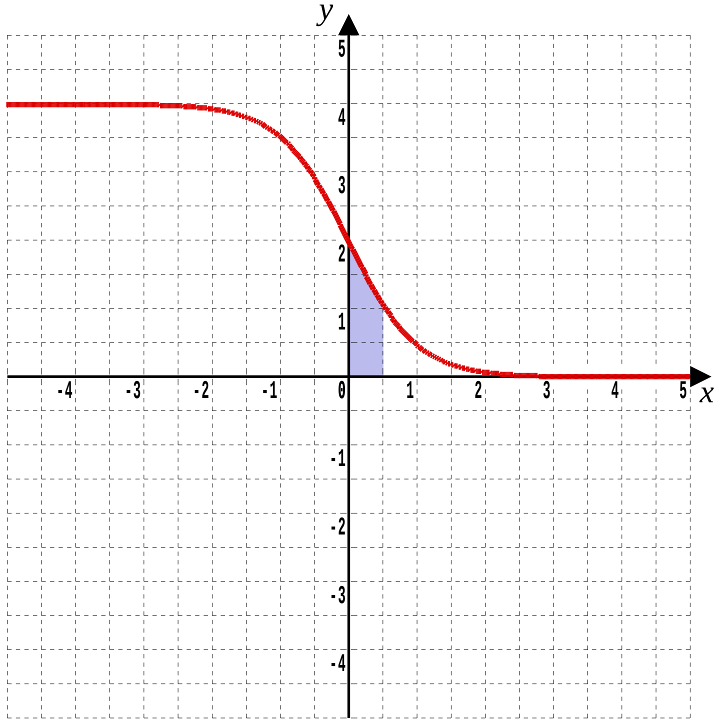
$$\theta \;=\; 0.253$$

Notice that the small angle approximations did a good job and got the correct answer to two decimal places.

[4 marks]

Answer 3

(i)



[1 mark]

(ii)

$$u = e^{2x} \Rightarrow \frac{du}{dx} = 2\,e^{2x}$$

$$\frac{du}{dx} = 2\,u$$

Change limits : when $x = 0,\ u = 1$

when $x = 0.5,\ u = e$

$$Area = \int_0^{0.5} \frac{4}{e^{2x} + 1}\,dx$$

$$= \int_1^e \frac{4}{u + 1}\;\frac{1}{2u}\,du$$

$$= \int_1^e \frac{2}{u\,(\,u + 1\,)}\,du \qquad \square$$

[3 marks]

(iii) By partial fractions

$$\frac{2}{u\,(u+1)} = \frac{A}{u} + \frac{B}{u+1}$$

$$2 = A\,(u+1) + Bu$$

$$A = 2 \text{ and } B = -2$$

$$Area = \int_1^e \frac{2}{u} - \frac{2}{u+1}\,du$$

$$= \left[\,2\,ln\,u - 2\,ln(u+1)\,\right]_1^e$$

use the rule of logs here to avoid a 2 becoming isolated

$$= \left[\,2\,ln\left(\frac{u}{u+1}\right)\right]_1^e$$

$$= \left[\,ln\left(\frac{u}{u+1}\right)^2\right]_1^e$$

$$= ln\left(\frac{e}{e+1}\right)^2 - ln\left(\frac{1}{2}\right)^2$$

$$= ln\left(\frac{e}{e+1}\right)^2 + ln\left(\frac{1}{2}\right)^{-2}$$

$$= ln\left(\frac{e}{e+1}\right)^2 + ln(2)^2$$

$$= ln\left(\frac{2e}{e+1}\right)^2 \qquad \square$$

[5 marks]

Answer 4

(i) As 2 is a root, replacing x with 2 will make the equation true (by the factor theorem)

$$(2)^3 + k(2) + 4 = 0$$

$$8 + 2k + 4 = 0$$

$$k = -6$$

(ii)

$$\begin{array}{r|l}
 & x^2 + 2x - 2 \\
\hline
x - 2 & x^3 + 0x^2 - 6x + 4 \\
 & \underline{x^3 - 2x^2} \\
 & + 2x^2 - 6x \\
 & \underline{+ 2x^2 - 4x} \\
 & - 2x + 4 \\
 & \underline{- 2x + 4} \\
 & 0
\end{array}$$

$$x^3 - 6x + 4 = (x - 2)(x^2 + 2x - 2)$$

for the quadratic factor...

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-2)}}{2 \times 1}$$

$$x = -1 \pm \sqrt{3}$$

[4 marks]