A-Level Pure Mathematics
Year 1, Year 2 Revision
Grade Grabber 3 Solutions

Answer 1

$$Area = \int_1^e 4x \ln(x)\ dx$$

$$= 4 \int_1^e \ln(x)\ \ x\ \ dx$$

$$\quad\quad \textbf{\textit{L}} \quad \textbf{\textit{I}} \quad\quad \textbf{\textit{D}} \quad \textbf{\textit{I}}$$

$$= 4 \left[\ln(x)\ \frac{x^2}{2} - \int_1^e \frac{1}{x} \times \frac{x^2}{2} \right]$$

$$= 4 \left[\frac{x^2\ \ln(x)}{2} - \frac{1}{2} \int_1^e x\ dx \right]$$

$$= 4 \left[\frac{x^2 \ln x}{2} - \frac{x^2}{4} \right]_1^e$$

$$= \left[2x^2 \ln x - x^2 \right]_1^e$$

$$= [\, 2e^2 - e^2 \,] - [\, 0 - 1 \,]$$

$$= e^2 + 1$$

[6 marks]

Answer 2

$$\sum_{n=0}^{\infty} x^{2n} = x^0 + x^2 + x^4 + x^6 + \ldots$$

This is a geometric progression with initial term 1 and common ratio x^2

The sum to infinity when $x^2 < 1$ exists and is given by;

$$Sum_\infty = \frac{a}{1 - r}$$

$$= \frac{1}{1 - x^2} \quad \square$$

[4 marks]

Answer 3

$$y = 8t + 1$$

$$y - 1 = 8t$$

$$t = \frac{y - 1}{8}$$

Substitution into $\quad x = 7 - \dfrac{3}{4t}$

gives $\quad x = 7 - \dfrac{3 \times 8}{4\,(y - 1)}$

$$x = 7 - \frac{24}{4\,(y - 1)}$$

$$x = 7 - \frac{6}{y - 1}$$

$$\frac{6}{y - 1} = 7 - x$$

$$\frac{y - 1}{6} = \frac{1}{7 - x}$$

$$y - 1 = \frac{6}{7 - x}$$

$$y = \frac{6}{7 - x} + 1$$

$$y = \frac{6}{7 - x} + \frac{7 - x}{7 - x}$$

$$y = \frac{13 - x}{7 - x} \qquad x \neq 7$$

So, $a = 13$, $b = 7$ and $c = 7$

[7 marks]

Answer 4

(i) Moving through the flowchart, left to right...

$$x \Rightarrow x + 1 \Rightarrow \frac{1}{x + 1} \Rightarrow -\frac{1}{x + 1} \Rightarrow -\frac{1}{x + 1} + 1$$

$$Now, \quad -\frac{1}{x + 1} + 1 = \frac{-1}{x + 1} + \frac{x + 1}{x + 1}$$

$$= \frac{x}{x + 1} \qquad x \neq -1 \qquad \square$$

[3 marks]

(ii)

$$f(x) = \frac{x}{x+1} \qquad x \neq -1$$

$$y = \frac{x}{x+1}$$

$$x = \frac{y}{y+1} \qquad \text{swapping input and output}$$

$$x(y+1) = y$$

$$xy + x = y$$

$$y - xy = x$$

$$y(1-x) = x$$

$$y = \frac{x}{1-x}$$

$$f^{-1}(x) = \frac{x}{1-x} \qquad x \neq 1$$

[4 marks]

(iii)

Reversing the flow through the flowchart...

$$f^{-1}(x) \longleftarrow \boxed{-1} \longleftarrow \boxed{\text{Reciprocal}} \longleftarrow \boxed{+/-} \longleftarrow \boxed{-1} \longleftarrow x$$

$$x \Rightarrow x - 1 \Rightarrow -(x-1) \Rightarrow \frac{1}{-(x-1)} \Rightarrow \frac{1}{-(x-1)} - 1$$

Now, $$\frac{1}{-(x-1)} - 1 = \frac{1}{1-x} - 1$$

$$= \frac{1}{1-x} - \frac{1-x}{1-x}$$

$$= \frac{1-(1-x)}{1-x}$$

$$= \frac{x}{1-x}$$

And so we have again that,

$$f^{-1}(x) = \frac{x}{1-x} \qquad x \neq 1$$

[6 marks]