

A-Level Pure Mathematics
Year 1, Year 2 Revision
Grade Grabber 4 Solutions

Answer 1

$$\begin{aligned}
 \text{(i)} \quad \frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} &= \frac{\cos^2 x + (1 - \sin x)^2}{(1 - \sin x) \cos x} \\
 &= \frac{\cos^2 x + 1 - 2 \sin x + \sin^2 x}{(1 - \sin x) \cos x} \\
 &= \frac{2 - 2 \sin x}{(1 - \sin x) \cos x} \\
 &= \frac{2(1 - \sin x)}{(1 - \sin x) \cos x} \\
 &= 2 \sec x \quad \square
 \end{aligned}$$

That is, $k = 2$

[4 marks]

$$\begin{aligned}
 \text{(ii)} \quad \frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} &= 0.8 \\
 2 \sec x &= 0.8 \\
 \frac{\cos x}{2} &= \frac{1}{0.8} \\
 \cos x &= 2.5
 \end{aligned}$$

This has no solutions as the maximum value of the cosine curve is 1

[2 marks]

Answer 2

$$\begin{aligned}
 y &= 5 \cos 2t \\
 &= 5 (\cos^2 t - \sin^2 t) \\
 &= 5 \cos^2 t - 5 \sin^2 t \\
 &= 5 (1 - \sin^2 t) - 5 \sin^2 t \\
 &= 5 - 10 \sin^2 t \\
 &= 5 - \frac{10 x^2}{6^2} \\
 &= 5 - \frac{10 x^2}{36} \\
 &= -\frac{5}{18} x^2 + 5
 \end{aligned}$$

That is, $a = -\frac{5}{18}$, $b = 0$ and $c = 5$

[4 marks]

Answer 3

$$\begin{aligned}ff(x) &= f\left(\frac{6x}{2x+3}\right) \\&= \frac{6\left(\frac{6x}{2x+3}\right)}{2\left(\frac{6x}{2x+3}\right)+3} \\&= \frac{36x}{12x+3(2x+3)} \\&= \frac{36x}{18x+9} \\&= \frac{36x}{9(2x+1)} \\&= \frac{4x}{2x+1} \quad \square\end{aligned}$$

[4 marks]

Answer 4

$$\frac{dy}{dx} = Kxy$$

$$\text{when } x = 1, y = 1 \text{ and } \frac{dy}{dx} = 4$$

$$4 = K \times 1 \times 1 \Rightarrow K = 4$$

$$\frac{dy}{dx} = 4xy$$

separate the variables

$$\frac{1}{y} \frac{dy}{dx} = 4x$$

integrate both sides with respect to x

$$\int \frac{1}{y} dy = \int 4x dx$$

$$\ln y = 2x^2 + c$$

$$y = e^{2x^2+c}$$

$$= e^{2x^2} e^c$$

$$= Ae^{2x^2}$$

$$\text{but } x = 1 \text{ when } y = 1 \Rightarrow 1 = Ae^2 \Rightarrow A = e^{-2}$$

$$y = e^{-2} e^{2x^2}$$

$$y = e^{2x^2-2}$$

[8 marks]

Answer 5**(i)**

$$\begin{aligned}\int y \, dx &= \int \tan x \sec^{16} x \, dx \\&= \int \tan x \sec x \sec^{15} x \, dx \\&= \int (\sec x \tan x) (\sec x)^{15} \, dx \\&= \frac{\sec^{16} x}{16} + c\end{aligned}$$

[4 marks]**(ii)**

$$\begin{aligned}\int y \, dx &= \int \tan x \sec^{16} x \, dx \\&= \int \frac{\sin x}{\cos x} \times \frac{1}{\cos^{16} x} \, dx \\&= (-1) \int (-\sin x) (\cos x)^{-17} \, dx \\&= (-1) \frac{\cos^{-16} x}{-16} + c \\&= \frac{1}{16 \cos^{16} x} + c \\&= \frac{\sec^{16} x}{16} + c\end{aligned}$$

[4 marks]