

A-Level Pure Mathematics
Year 1, Year 2 Revision
Grade Grabber 5 Solutions

Answer 1

$$\frac{dy}{dx} = \frac{K}{xy}$$

when $x = 1$, $y = 1$ and $\frac{dy}{dx} = 7$

$$7 = \frac{K}{1 \times 1}$$

$$K = 7$$

$$\frac{dy}{dx} = \frac{7}{xy}$$

separate the variables

$$y \frac{dy}{dx} = \frac{7}{x}$$

integrate both sides with respect to x

$$\int y \, dy = 7 \int \frac{1}{x} \, dx$$

$$\frac{y^2}{2} = 7 \ln|x| + c$$

$$y^2 = 14 \ln|x| + A$$

but $x = 1$ when $y = 1$

$$1 = 14 \ln 1 + A$$

$$\therefore A = 1$$

$$y^2 = 14 \ln|x| + 1$$

[7 marks]

Answer 2

$$\begin{aligned}\text{(i)} \quad \overrightarrow{OS} &= \overrightarrow{OP} + \overrightarrow{PS} \\ &= \begin{pmatrix} 7 \\ 12 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -11 \\ 16 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ 1 \\ 15 \end{pmatrix}\end{aligned}$$

[2 marks]

$$\begin{aligned}\text{(ii)} \quad \overrightarrow{OR} &= \overrightarrow{OQ} + \overrightarrow{QR} \\ &= \begin{pmatrix} 11 \\ 2 \\ a \end{pmatrix} + \begin{pmatrix} 3 \\ -16 \\ 12 \end{pmatrix} \\ &= \begin{pmatrix} 14 \\ -14 \\ a + 12 \end{pmatrix}\end{aligned}$$

[2 marks]

$$\begin{aligned}\text{(iii)} \quad \overrightarrow{SR} &= \overrightarrow{SO} + \overrightarrow{OR} \\ &= \begin{pmatrix} -8 \\ -1 \\ -15 \end{pmatrix} + \begin{pmatrix} 14 \\ -14 \\ a + 12 \end{pmatrix} \\ &= \begin{pmatrix} 6 \\ -15 \\ a - 3 \end{pmatrix}\end{aligned}$$

[2 marks]

$$\begin{aligned}\text{(iv)} \quad \overrightarrow{PQ} &= \overrightarrow{PO} + \overrightarrow{OQ} \\ &= \begin{pmatrix} -7 \\ -12 \\ 1 \end{pmatrix} + \begin{pmatrix} 11 \\ 2 \\ a \end{pmatrix} \\ &= \begin{pmatrix} 4 \\ -10 \\ 1 + a \end{pmatrix}\end{aligned}$$

[2 marks]

$$\begin{aligned}\text{(v)} \quad \overrightarrow{SR} &= 1.5 \overrightarrow{PQ} \\ \therefore a - 3 &= 1.5 (1 + a) \\ a - 3 &= 1.5 + 1.5a \\ a &= -9\end{aligned}$$

[3 marks]

Answer 3

$$(i) \quad \cos^2 \theta + \sin^2 \theta = 1$$

$$\cos^2 \theta - \sin^2 \theta = \cos 2\theta$$

[2 marks]

$$(ii) \quad 2 \cos^2 \theta = 1 + \cos 2\theta$$

$$\cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$$

[2 marks]

$$(iii) \quad \int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{2} + \frac{1}{2} \cos 2\theta \, d\theta$$

$$= \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{4}$$

[3 marks]**Answer 4**

$$x = -3 \cot y$$

$$\frac{dx}{dy} = 3 \csc^2 y$$

$$= 3 (\cot^2 y + 1)$$

$$= 3 \left(\left(-\frac{x}{3} \right)^2 + 1 \right)$$

$$= 3 \left(\frac{x^2}{9} + \frac{9}{9} \right)$$

$$= \frac{3}{9} (x^2 + 9)$$

$$= \frac{x^2 + 9}{3}$$

$$\frac{dy}{dx} = \frac{3}{x^2 + 9}$$

Thus $a = 3$ and $b = 9$ **[5 marks]**