GRADE GRABBER

Numbers 1 to 5

(With answers)

A-Level Pure Mathematics

For Summer 2025

* REVISION *

A-Level Pure Mathematics
Year 1, Year 2 Revision

Grade Grabber 1

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 34

Question 1

Given that

$$\sum_{m=1}^{k} (7m + 3) = 777$$

determine the value of the integer constant k

[4 marks]

Question 2

Sid claims that it's always true that

$$x^2 < (x + 1)^2$$

Exhibit a counter-example that shows Sid's claim is not true

[2 marks]

Question 3

The equation

$$5x^2 + k = 3x + 7$$

has two distinct real roots.

Find the range of possible values for k.

[4 marks]

Question 4

A circle of radius r cm is divided by two radii into two sectors.

The smaller sector has perimeter $(2r + 4)$ cm and the larger sector has area 12 cm^2.

Calculate the value or r correct to 3 decimal places.

[6 marks]

Question 5

(i) Use the binomial theorem to write $\dfrac{1}{(1 - x)^3}$ as a polynomial, in ascending powers of x, up to and including the term in x^3

[4 marks]

(ii) By spotting a connection with the triangular numbers, state the coefficient in the expansion of x^8

[2 marks]

Question 6

Given that

$$f(x) = e^{5x} - 1, \qquad x \in \mathbb{R}$$

find $f^{-1}(x)$ and state its domain.

[4 marks]

Question 7

With respect to a fixed origin O, the points A and B have coordinates $(-4, 0)$ and $(12, 12)$ respectively.

The mid-point of AB is M.

Find an equation of the line in the plane of the coordinate axes Ox and Oy which passes through M and is perpendicular to AB.

Hence, or otherwise, find, in cartesian form, an equation of the circle which passes through O, A and B.

[8 marks]

A-Level Pure Mathematics
Year 1, Year 2 Revision
Grade Grabber 1 Solutions

Answer 1

$$\sum_{m=1}^{k} (7m + 3) = 777$$

$$10 + 17 + 24 + \ldots + (7k + 3) = 777$$

The sequence is is Arithmetic Progression; $a = 10, \quad d = 7, \quad n = k$

$$Sum = \frac{n}{2}\{2a + (n - 1)d\}$$

$$\frac{k}{2}\{20 + (k - 1)7\} = 777$$

$$20k + 7k^2 - 7k = 1554$$

$$7k^2 + 13k - 1554 = 0$$

$$k = \frac{-13 \pm \sqrt{13^2 - 4 \times 7 \times (-1554)}}{2 \times 7}$$

$$= 14 \quad \text{or} \quad -\frac{111}{7}$$

$$\therefore k = 14$$

[4 marks]

Answer 2

Let $x = -1$: Then $x^2 = 1$ and $(x + 1)^2 = 0$

This makes the given statement False ☐

In fact, any $x < -0.5$ makes the given statement false

[2 marks]

Answer 3

$$5x^2 + k = 3x + 7$$

$$5x^2 - 3x + k - 7 = 0$$

To have two distinct real roots the Discriminant must be positive

$$D > 0$$

$$b^2 - 4ac > 0$$

$$(-3)^2 - 4 \times 5 \times (k - 7) > 0$$

$$9 - 20k + 140 > 0$$

$$20k < 149$$

$$k < 7.45$$

[4 marks]

Answer 4

$$Arc\ Length\ :\ r\theta = 4$$

$$Sector\ Area\ :\ \frac{1}{2} r^2 \ (2\pi - \theta) = 12$$

$$\text{Combining by substitution yields}\ :\ \frac{1}{2} r^2 \left(2\pi - \frac{4}{r} \right) = 12$$

$$\pi r^2 - 2r - 12 = 0$$

$$r = \frac{2 \pm \sqrt{2^2 - 4\pi(-12)}}{2\pi}$$

$$r = 2.298 \quad \text{or} - 1.662$$

$$\therefore\ r = 2.298 \text{ cm}$$

[6 marks]

Answer 5

(i) The binomial theorem states;

$$(1 + x)^n = 1 + nx + \frac{n (n - 1)}{2 !} x^2 + \frac{n (n - 1) (n - 2)}{3 !} x^3 + \ ...$$

... with $n = -3$ and x replaced with $(-x)$ yields...

$$(1 - x)^{-3} = 1 + \left(-3\right)(-x) + \frac{(-3)(-4)}{2} (-x)^2 + \frac{(-3)(-4)(-5)}{6} (-x)^3$$

$$= 1 + 3x + 6x^2 + 10x^3 + \ ... \qquad \text{valid for } \left| x \right| < 1$$

[4 marks]

(ii) The coefficients are the triangular numbers....

$$(1 - x)^{-3}$$

$$= 1 + 3x + 6x^2 + 10x^3 + 15x^4 + 21x^5 + 28x^6 + 36x^7 + 45x^8 + \ ...$$

$$\therefore \text{ Coefficient of } x^8 \text{ is } 45$$

[2 marks]

Answer 6

$$y = e^{5x} - 1$$

$$x = e^{5y} - 1 \qquad \text{swapping input and output}$$

$$x + 1 = e^{5y}$$

$$ln(x + 1) = 5y$$

$$y = \frac{1}{5} ln(x + 1)$$

$$f^{-1}(x) = \frac{1}{5} ln(x + 1) \qquad x \in \mathbb{R},\ x > -1 \ \Leftarrow \ \text{Domian}$$

[4 marks]

Answer 7

$$M = \left(\frac{-4 + 12}{2}, \frac{0 + 12}{2} \right)$$

$$= (4. 6)$$

$$m_{AB} = \frac{12 - 0}{12 - (-4)}$$

$$= \frac{3}{4}$$

$$\therefore m_{pPERP} = -\frac{4}{3}$$

AB will have a perpendicular bisector with an equation of the form $y = mx + c$

$$6 = -\frac{4}{3} \times 4 + c$$

$$c = \frac{34}{3}$$

$$y = -\frac{4}{3}x + \frac{34}{3}$$

Strategy

The strategy is now to repeat the above process.
Pick two more points on the circumference of the circle, then get the perpendicular bisector of the line segment between those two points.

The centre of the circle is then where the two different perpendicular bisectors intersect.

Incidently, this is how you would physically do the question with a straight edge pencil and compass !

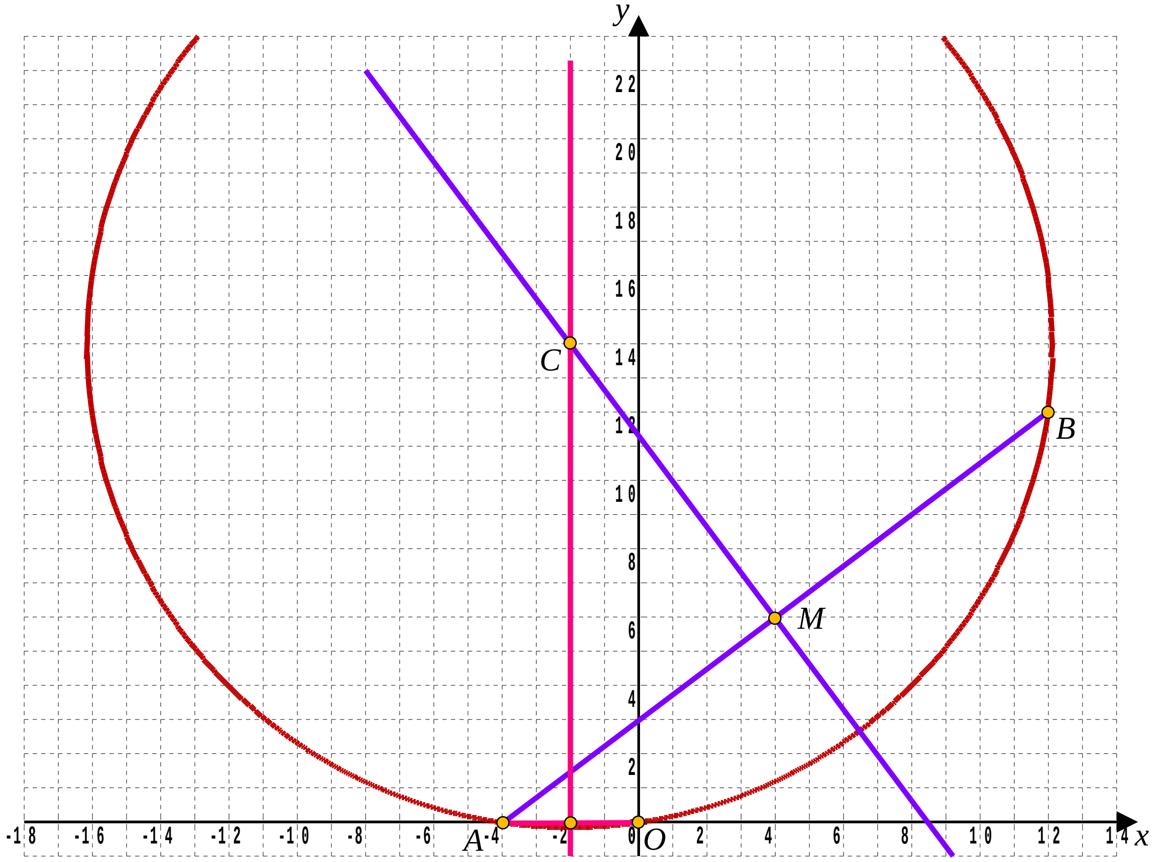
There is a clever way of getting the x-coordinate of the circle's centre using the fact that the circumference passes through both $A(-4, 0)$ and $O(0, 0)$ both on the x-axis, so the perpendicular bisector of the line segment between those two points will be the vertical line with equation $x = -2$.
As the circle centre must be somewhere on that perpendicular bisector the circle centre has $x = -2$

To find where $x = -2$ and $y = -\frac{4}{3}x + \frac{34}{3}$ intersect,

$$y = -\frac{4}{3}(-2) + \frac{34}{3}$$

$$y = 14$$

Here is a diagram of the situation,



Use Pythagoras' Theorem to get the radius: $\sqrt{(-2)^2 + 14^2} = \sqrt{200}$

The equation of the circle is thus: $(x + 2)^2 + (y - 14)^2 = 200$

[8 marks]

A-Level Pure Mathematics
Year 1, Year 2 Revision

Grade Grabber 2

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 26

Question 1

A geometric series has first term $\sin 2\theta$ and common ratio $\cos 2\theta$

where $0 < \theta < \dfrac{\pi}{4}$

Show that the sum to infinity of this series is $\cot \theta$

[4 marks]

Question 2
Consider the equation;

$$13 \sin\theta + 8\cos\theta = 11 \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

This question is about solving this equation in two different ways, one approximate, the other exact.

(a) You are reminded that if θ is small and measured in radians,

- $\sin\theta \approx \theta$
- $\cos\theta \approx 1 - \dfrac{\theta^2}{2}$
- $\tan\theta \approx \theta$

Use the small angle approximations to show that;

$$4\theta^2 - 13\theta + 3 = 0$$

and hence find a valid approximate solutions to

$$13 \sin\theta + 8\cos\theta = 11 \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

[4 marks]

(b) Assume that

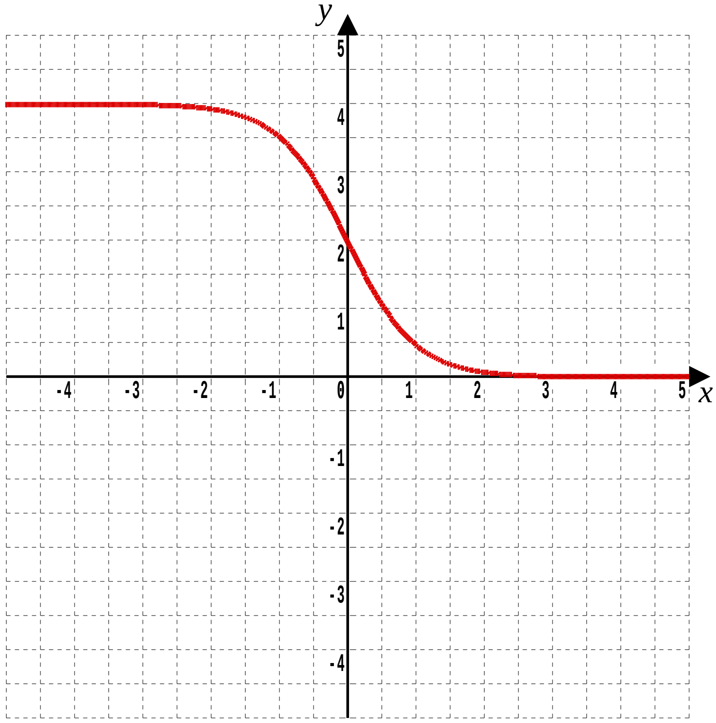
$$13 \sin \theta + 8 \cos \theta = R \sin (\theta + \alpha) \qquad 0 \leqslant \alpha \leqslant \frac{\pi}{2}$$

Determine the value of α and the value of R and hence solve

$$13 \sin \theta + 8 \cos \theta = 11 \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

[4 marks]

Question 3



The graph shows a plot of the curve with equation

$$y = \frac{4}{e^{2x} + 1} \qquad x \in R$$

The area of a finite region, bounded by the curve, the y-axis, the x-axis, and the line with equation $x = 0.5$ is of interest.

(i) On the diagram, shade in the area of interest.

[1 mark]

(ii) Use the substitution $u = e^{2x}$ to show that the area of interest can be given by

$$\int_a^b \frac{2}{u\,(\,u + 1\,)}\,du$$

where a and b are constants to be found

[3 marks]

(iii) Hence use algebraic integration to show that the exact area of the region of interest is

$$Area = ln\left(\frac{2e}{e+1}\right)^2$$

[5 marks]

Question 4

One root of the following equation is 2;

$$x^3 + kx + 4 = 0$$

(i) Find the value of k

[1 mark]

(ii) Find the exact values of the other two roots.

[4 marks]

A-Level Pure Mathematics
Year 1, Year 2 Revision
Grade Grabber 2 Solutions

Answer 1

For a geometric progression

$$\begin{aligned}
Sum_{\infty} &= \frac{a}{1 - r} \\
&= \frac{\sin 2\theta}{1 - \cos 2\theta} \\
&= \frac{2 \sin\theta \cos\theta}{\cos^2\theta + \sin^2\theta - (\cos^2\theta - \sin^2\theta)} \\
&= \frac{2 \sin\theta \cos\theta}{2 \sin^2\theta} \\
&= \frac{\cos\theta}{\sin\theta} \\
&= \cot\theta \qquad \square
\end{aligned}$$

[4 marks]

Answer 2
(a)

$$13\,sin\,\theta \;+\; 8\,cos\,\theta \;=\; 11$$

$$13\theta \;+\; 8\left(1 \;-\; \frac{\theta^2}{2}\right) \;=\; 11 \qquad \text{using small angle aooroximations}$$

$$13\theta \;+\; 8 \;-\; 4\,\theta^2 \;=\; 11$$

$$4\,\theta^2 \;-\; 13\theta \;+\; 3 \;=\; 0 \qquad \square$$

$$(\,4\theta \;-\; 1\,)(\,\theta \;-\; 3\,) \;=\; 0$$

$$\therefore\;\; \theta \;\in\; \{\,0.25,\;\; 3\}$$

The angle of 3 radians is too large to be a small angle approximation and so is rejected leaving $\theta = 0.25$ radians as the only solution.

[4 marks]

(b)

$$R\,sin\;(\,\theta \;+\; \alpha\,) \;=\; R\,sin\,\theta\,cos\,\alpha \;+\; R\,cos\,\theta\,sin\,\alpha$$

Comparison with $13\,sin\,\theta \;+\; 8\,cos\,\theta$ yields;

$$\frac{R\,sin\,\alpha}{R\,cos\,\alpha} \;=\; \frac{8}{13}$$

$$\alpha \;=\; arctan\left(\frac{8}{13}\right)$$

$$\alpha \;=\; 0.5517^c$$

$$R \;=\; \sqrt{8^2 \;+\; 13^2}$$

$$=\; 15.264$$

Thus,

$$13\,sin\,\theta \;+\; 8\,cos\,\theta \;=\; 11$$

$$15.264\,sin\;(\,\theta \;+\; 0.5517\,) \;=\; 11$$

$$sin\;(\,\theta \;+\; 0.5517\,) \;=\; 0.7206$$

$$\theta \;+\; 0.5517 \;=\; arcsin\,0.7206$$

$$\theta \;+\; 0.5517 \;=\; 0.8047$$

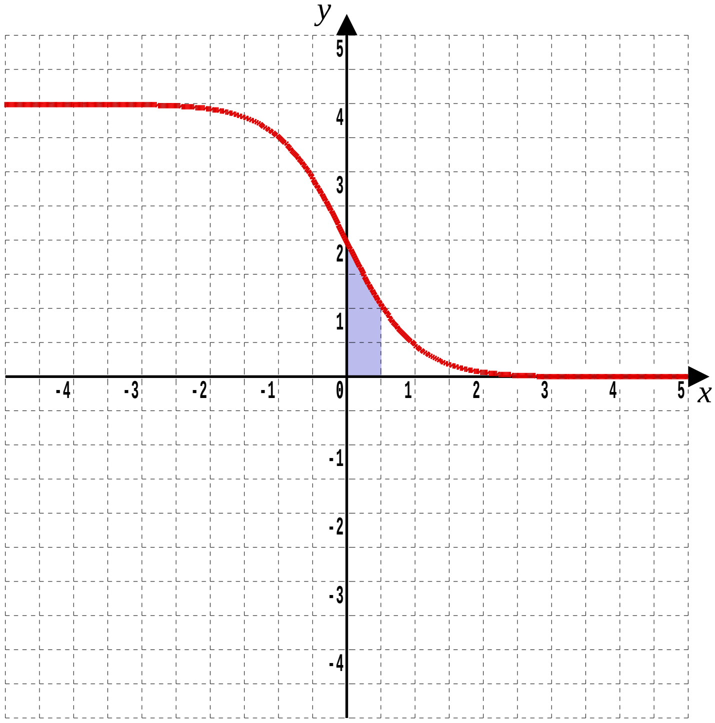
$$\theta \;=\; 0.253$$

Notice that the small angle approximations did a good job and got the correct answer to two decimal places.

[4 marks]

Answer 3

(i)



[1 mark]

(ii)

$$u = e^{2x} \Rightarrow \frac{du}{dx} = 2\,e^{2x}$$

$$\frac{du}{dx} = 2\,u$$

Change limits : when $x = 0, \; u = 1$

when $x = 0.5, \; u = e$

$$Area = \int_0^{0.5} \frac{4}{e^{2x} + 1}\, dx$$

$$= \int_1^e \frac{4}{u + 1} \; \frac{1}{2u}\, du$$

$$= \int_1^e \frac{2}{u\,(u + 1)}\, du \qquad \square$$

[3 marks]

(iii) By partial fractions

$$\frac{2}{u\,(u+1)} = \frac{A}{u} + \frac{B}{u+1}$$

$$2 = A\,(u+1) + Bu$$

$$A = 2 \text{ and } B = -2$$

$$Area = \int_1^e \frac{2}{u} - \frac{2}{u+1}\,du$$

$$= \left[\,2\,ln\,u - 2\,ln(u+1)\,\right]_1^e$$

use the rule of logs here to avoid a 2 becoming isolated

$$= \left[\,2\,ln\left(\frac{u}{u+1}\right)\right]_1^e$$

$$= \left[\,ln\left(\frac{u}{u+1}\right)^2\,\right]_1^e$$

$$= ln\left(\frac{e}{e+1}\right)^2 - ln\left(\frac{1}{2}\right)^2$$

$$= ln\left(\frac{e}{e+1}\right)^2 + ln\left(\frac{1}{2}\right)^{-2}$$

$$= ln\left(\frac{e}{e+1}\right)^2 + ln(2)^2$$

$$= ln\left(\frac{2e}{e+1}\right)^2 \qquad \square$$

[5 marks]

Answer 4

(i) As 2 is a root, replacing x with 2 will make the equation true (by the factor theorem)

$$(2)^3 + k(2) + 4 = 0$$

$$8 + 2k + 4 = 0$$

$$k = -6$$

(ii)

$$\begin{array}{r|l}
 & \quad\quad\;\; x^2 + 2x - 2 \\
\hline
x - 2 & x^3 + 0x^2 - 6x + 4 \\
 & x^3 - 2x^2 \\
\cline{2-2}
 & \quad\;\; + 2x^2 - 6x \\
 & \quad\;\; + 2x^2 - 4x \\
\cline{2-2}
 & \quad\quad\quad\;\; - 2x + 4 \\
 & \quad\quad\quad\;\; - 2x + 4 \\
\cline{2-2}
 & \quad\quad\quad\quad\quad\quad\; 0
\end{array}$$

$$x^3 - 6x + 4 = (x - 2)(x^2 + 2x - 2)$$

for the quadratic factor...

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-2)}}{2 \times 1}$$

$$x = -1 \pm \sqrt{3}$$

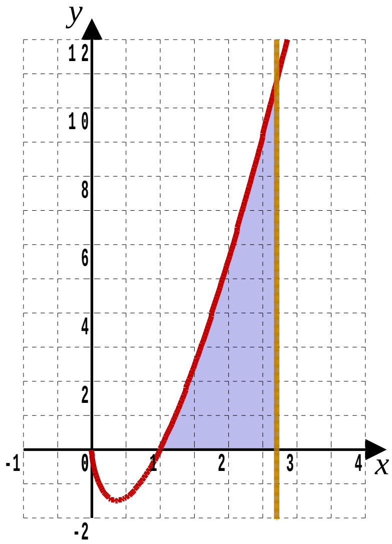
[4 marks]

A-Level Pure Mathematics
Year 1, Year 2 Revision

Grade Grabber 3

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 30

Question 1



The graph is of the curve $y = 4x \ln x$. and the vertical line $x = e$
Find the exact area bounded by the curve, the x-axis, and the lines, $x = 1$ and $x = e$

[6 marks]

Question 2

Prove that

$$\sum_{n=0}^{\infty} x^{2n} = \frac{1}{1 - x^2}$$

[4 marks]

Question 3

A curve has parametric equations

$$x = 7 - \frac{3}{4t} \qquad y = 8t + 1$$

Show that the Cartesian equation of the curve can be expressed in the form

$$y = \frac{a - x}{b - x} \qquad x \neq c$$

where a, b and c are constants to be found

[7 marks]

Question 4

The flowchart describes a function f

$$x \rightarrow \boxed{+\,1} \rightarrow \boxed{\text{Reciprocal}} \rightarrow \boxed{+/-} \rightarrow \boxed{+\,1} \rightarrow f(x)$$

(i) Show that the function described can be written as

$$f(x) = \frac{x}{x+1} \qquad x \neq -1$$

[3 marks]

(ii) Use algebraic manipulation to find $f^{-1}(x)$

[4 marks]

The task now is to use the flowchart of f to obtain your part (ii) answer.

$$x \longrightarrow \boxed{+\,1} \longrightarrow \boxed{\text{Reciprocal}} \longrightarrow \boxed{+/-} \longrightarrow \boxed{+\,1} \longrightarrow f(x)$$

(iii) Explain your strategy, drawing an appropriately modified flowchart, and so verify your part (ii) answer from this alternative method.

[6 marks]

A-Level Pure Mathematics
Year 1, Year 2 Revision
Grade Grabber 3 Solutions

Answer 1

$$Area = \int_1^e 4x \, ln(x) \; dx$$

$$= 4 \int_1^e ln(x) \;\; x \;\; dx$$

$$\qquad \textbf{\textit{L}} \quad \textbf{\textit{I}} \qquad \textbf{\textit{D}} \quad \textbf{\textit{I}}$$

$$= 4 \left[\, ln(x) \;\; \frac{x^2}{2} - \int_1^e \frac{1}{x} \times \frac{x^2}{2} \, \right]$$

$$= 4 \left[\, \frac{x^2 \;\, ln(x)}{2} \;\; - \frac{1}{2} \int_1^e x \;\, dx \, \right]$$

$$= 4 \left[\, \frac{x^2 \, ln \, x}{2} - \frac{x^2}{4} \, \right]_1^e$$

$$= \left[\, 2 \, x^2 \, ln \, x - x^2 \, \right]_1^e$$

$$= [\, 2 \, e^2 - e^2 \,] - [\, 0 - 1 \,]$$

$$= e^2 + 1$$

[6 marks]

Answer 2

$$\sum_{n=0}^{\infty} x^{2n} = x^0 + x^2 + x^4 + x^6 + \ldots$$

This is a geometric progression with initial term 1 and common ratio x^2

The sum to infinity when $x^2 < 1$ exists and is given by;

$$Sum_\infty = \frac{a}{1 - r}$$

$$= \frac{1}{1 - x^2} \qquad \square$$

[4 marks]

Answer 3

$$y = 8t + 1$$

$$y - 1 = 8t$$

$$t = \frac{y-1}{8}$$

Substitution into $\quad x = 7 - \dfrac{3}{4t}$

gives $\quad x = 7 - \dfrac{3 \times 8}{4\,(y-1)}$

$$x = 7 - \frac{24}{4\,(y-1)}$$

$$x = 7 - \frac{6}{y-1}$$

$$\frac{6}{y-1} = 7 - x$$

$$\frac{y-1}{6} = \frac{1}{7-x}$$

$$y - 1 = \frac{6}{7-x}$$

$$y = \frac{6}{7-x} + 1$$

$$y = \frac{6}{7-x} + \frac{7-x}{7-x}$$

$$y = \frac{13-x}{7-x} \qquad x \neq 7$$

So, $a = 13$, $b = 7$ and $c = 7$

[7 marks]

Answer 4

(i) Moving through the flowchart, left to right...

$$x \Rightarrow x + 1 \Rightarrow \frac{1}{x+1} \Rightarrow -\frac{1}{x+1} \Rightarrow -\frac{1}{x+1} + 1$$

$$Now, \quad -\frac{1}{x+1} + 1 = \frac{-1}{x+1} + \frac{x+1}{x+1}$$

$$= \frac{x}{x+1} \qquad x \neq -1 \qquad \Box$$

[3 marks]

(ii)

$$f\,(x) \;=\; \frac{x}{x + 1} \qquad x \neq -1$$

$$y \;=\; \frac{x}{x + 1}$$

$$x \;=\; \frac{y}{y + 1} \qquad \text{swapping input and output}$$

$$x\,(\,y + 1\,) \;=\; y$$

$$xy + x \;=\; y$$

$$y - xy \;=\; x$$

$$y\,(\,1 - x\,) \;=\; x$$

$$y \;=\; \frac{x}{1 - x}$$

$$f^{\,-1}(x) \;=\; \frac{x}{1 - x} \qquad x \neq 1$$

[4 marks]

(iii)

Reversing the flow through the flowchart...

$$f^{\,-1}(x) \longleftarrow \boxed{-\,1} \longleftarrow \boxed{\text{Reciprocal}} \longleftarrow \boxed{+/-} \longleftarrow \boxed{-\,1} \longleftarrow x$$

$$x \;\Rightarrow\; x - 1 \;\Rightarrow\; -(\,x - 1\,) \;\Rightarrow\; \frac{1}{-\,(\,x - 1\,)} \;\Rightarrow\; \frac{1}{-\,(\,x - 1\,)} - 1$$

Now,
$$\frac{1}{-\,(\,x - 1\,)} - 1 \;=\; \frac{1}{1 - x} - 1$$
$$=\; \frac{1}{1 - x} - \frac{1 - x}{1 - x}$$
$$=\; \frac{1 - (\,1 - x\,)}{1 - x}$$
$$=\; \frac{x}{1 - x}$$

And so we have again that,

$$f^{\,-1}(x) \;=\; \frac{x}{1 - x} \qquad x \neq 1$$

[6 marks]

A-Level Pure Mathematics
Year 1, Year 2 Revision

Grade Grabber 4

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 30

Question 1

(i) Show that

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = k \sec x \qquad x \neq \frac{n\pi}{2},\ n \in \mathbb{Z}$$

where k is a constant to be found

[4 marks]

(ii) Hence explain why the equation

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = 0.8$$

has no real solutions

[2 marks]

Question 2

A curve has parametric equations $x = 6 \sin t \qquad y = 5 \cos 2t$

Find the Cartesian equation of the curve in the form $y = a x^2 + bx + c$ where a, b and c are real number constants.

[4 marks]

Question 3

The function f is defined by

$$f(x) = \frac{6x}{2x + 3} \qquad x \in \mathbb{R},\ x \geqslant 0$$

Show that

$$ff(x) = \frac{4x}{2x + 1}$$

[4 marks]

Question 4
The gradient of a curve at any point (x, y) on the curve is directly proportional to the product of x and y. The curve passes through the point (1, 1) and at this point the gradient of the curve is 4.

Form a differential equation in x and y and solve this equation to express y in terms of x

[8 marks]

Question 5

$$y = \tan x \ \sec^{16} x$$

This question is about finding the integral of y in two different ways.

(i) Use the fact that

$$\int y \, dx = \int (\sec x \ \tan x) \ (\sec x)^{15} \ dx$$

and “the chain rule backwards” to determine an algebraic answer for the integration

[4 marks]

(ii) Show that

$$\int y \, dx = (-1) \int (-\sin x) \ (\cos x)^{-17} \ dx$$

and hence, by means of “the chain rule backwards”, determine an algebraic answer for the integration. Manipulate your anwers to show that it is the same as that obtained in part (i).

[4 marks]

A-Level Pure Mathematics
Year 1, Year 2 Revision
Grade Grabber 4 Solutions

Answer 1

(i) $$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = \frac{\cos^2 x + (1 - \sin x)^2}{(1 - \sin x)\cos x}$$

$$= \frac{\cos^2 x + 1 - 2\sin x + \sin^2 x}{(1 - \sin x)\cos x}$$

$$= \frac{2 - 2\sin x}{(1 - \sin x)\cos x}$$

$$= \frac{2(1 - \sin x)}{(1 - \sin x)\cos x}$$

$$= 2\sec x \qquad \square$$

That is, $k = 2$

[4 marks]

(ii) $$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x} = 0.8$$

$$2\sec x = 0.8$$

$$\frac{\cos x}{2} = \frac{1}{0.8}$$

$$\cos x = 2.5$$

This has no solutions as the maximum value of the cosine curve is 1

[2 marks]

Answer 2

$$y = 5\cos 2t$$

$$= 5(\cos^2 t - \sin^2 t)$$

$$= 5\cos^2 t - 5\sin^2 t$$

$$= 5(1 - \sin^2 t) - 5\sin^2 t$$

$$= 5 - 10\sin^2 t$$

$$= 5 - \frac{10x^2}{6^2}$$

$$= 5 - \frac{10x^2}{36}$$

$$= -\frac{5}{18}x^2 + 5$$

That is, $a = -\frac{5}{18}$, $b = 0$ and $c = 5$

[4 marks]

Answer 3

$$ff(x) = f\left(\frac{6x}{2x+3}\right)$$

$$= \frac{6\left(\frac{6x}{2x+3}\right)}{2\left(\frac{6x}{2x+3}\right)+3}$$

$$= \frac{36x}{12x + 3(2x+3)}$$

$$= \frac{36x}{18x+9}$$

$$= \frac{36\,x}{9(2x+1)}$$

$$= \frac{4x}{2x+1} \qquad \square$$

[4 marks]

Answer 4

$$\frac{dy}{dx} = Kxy$$

when $x = 1,\ y = 1$ and $\frac{dy}{dx} = 4$

$$4 = K \times 1 \times 1 \Rightarrow K = 4$$

$$\frac{dy}{dx} = 4xy$$

separate the variables

$$\frac{1}{y}\frac{dy}{dx} = 4x$$

integrate both sides with respect to x

$$\int \frac{1}{y}\,dy = \int 4x\,dx$$

$$\ln y = 2x^2 + c$$

$$y = e^{2x^2+c}$$

$$= e^{2x^2}e^c$$

$$= Ae^{2x^2}$$

but $x = 1$ when $y = 1 \Rightarrow 1 = Ae^2 \Rightarrow A = e^{-2}$

$$y = e^{-2}e^{2x^2}$$

$$y = e^{2x^2-2}$$

[8 marks]

Answer 5
(i)

$$\int y\,dx = \int tan\,x\;\; sec^{16}\,x\;\; dx$$

$$= \int\; tan\,x\;\; sec\,x\;\;\; sec^{15}\,x\;\; dx$$

$$= \int\; (\; sec\,x\; tan\,x\;)\;\; (\; sec\,x\;)^{15}\;\; dx$$

$$= \frac{sec^{16}\,x}{16} \;+\; c$$

[4 marks]

(ii)

$$\int y\,dx = \int tan\,x\;\; sec^{16}\,x\;\; dx$$

$$= \int \frac{sin\,x}{cos\,x} \times \frac{1}{cos^{16}\,x}\; dx$$

$$= (-1)\int\; (-\,sin\,x)\;\; (cos\,x)^{-17}\;\; dx$$

$$= (-1)\;\frac{cos^{-16}\,x}{-16} \;+\; c$$

$$= \frac{1}{16\,cos^{16}\,x} \;+\; c$$

$$= \frac{sec^{16}\,x}{16} \;+\; c$$

[4 marks]

A-Level Pure Mathematics
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Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 30

Question 1

The gradient of a curve at any point (x, y) on the curve is inversely proportional to the product of x and y. The curve passes through the point (1, 1) and at this point the gradient of the curve is 7.

Form a differential equation in x and y and solve this equation to expess y^2 in terms of x

[7 marks]

Question 2

Relative to a fixed origin O, the points P and Q are such that

$$\overrightarrow{OP} = \begin{pmatrix} 7 \\ 12 \\ -1 \end{pmatrix} \qquad \overrightarrow{OQ} = \begin{pmatrix} 11 \\ 2 \\ a \end{pmatrix}$$

where p is a constant, and the points R and S are such that

$$\overrightarrow{QR} = \begin{pmatrix} 3 \\ -16 \\ 12 \end{pmatrix} \qquad \overrightarrow{PS} = \begin{pmatrix} 1 \\ -11 \\ 16 \end{pmatrix}$$

(i) Find the position vector of the point S

[2 marks]

(ii) Find $\overrightarrow{OR}$ in term of a

[2 marks]

(iii) Find $\overrightarrow{SR}$ in term of a

[2 marks]

(iv) Find $\overrightarrow{PQ}$ in term of a

[2 marks]

(v) Given that $\overrightarrow{SR}$ is parallel to $\overrightarrow{PQ}$, find the value of a

[3 marks]

Question 3

Two trigonometry identities involving $cos^2\theta$ and $sin^2\theta$ are;

$$cos^2\theta + sin^2\theta = p$$

$$cos^2\theta - sin^2\theta = q$$

where p is a constant and q is a trigonometric function in θ

(i) Write out the two identities with p and q replaced appropriately

[2 marks]

(ii) Treating the two identities as simultaneous equations, combine them by addition to obtain an expression for $cos^2\theta$

[2 mark]

(iii) Hence, or otherwise, evaluate

$$\int_0^{\frac{\pi}{2}} cos^2\theta \; d\theta$$

[3 marks]

Question 4
Given that

$$x = -3 \cot y \qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

show that

$$\frac{dy}{dx} = \frac{a}{x^2 + b}$$

where a and b are integer constants to be found

[5 marks]

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Answer 1

$$\frac{dy}{dx} = \frac{K}{xy}$$

$$\text{when } x = 1, \ y = 1 \text{ and } \frac{dy}{dx} = 7$$

$$7 = \frac{K}{1 \times 1}$$

$$K = 7$$

$$\frac{dy}{dx} = \frac{7}{xy}$$

separate the variables

$$y \frac{dy}{dx} = \frac{7}{x}$$

integrate both sides with respect to x

$$\int y \ dy = 7 \int \frac{1}{x} \ dx$$

$$\frac{y^2}{2} = 7 \ln|x| + c$$

$$y^2 = 14 \ln|x| + A$$

$$\text{but } x = 1 \text{ when } y = 1$$

$$1 = 14 \ln 1 + A$$

$$\therefore A = 1$$

$$y^2 = 14 \ln|x| + 1$$

[7 marks]

Answer 2

(i)

$$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS}$$

$$= \begin{pmatrix} 7 \\ 12 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ -11 \\ 16 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 1 \\ 15 \end{pmatrix}$$

[2 marks]

(ii)

$$\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR}$$

$$= \begin{pmatrix} 11 \\ 2 \\ a \end{pmatrix} + \begin{pmatrix} 3 \\ -16 \\ 12 \end{pmatrix}$$

$$= \begin{pmatrix} 14 \\ -14 \\ a + 12 \end{pmatrix}$$

[2 marks]

(iii)

$$\overrightarrow{SR} = \overrightarrow{SO} + \overrightarrow{OR}$$

$$= \begin{pmatrix} -8 \\ -1 \\ -15 \end{pmatrix} + \begin{pmatrix} 14 \\ -14 \\ a + 12 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -15 \\ a - 3 \end{pmatrix}$$

[2 marks]

(iv)

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$$

$$= \begin{pmatrix} -7 \\ -12 \\ 1 \end{pmatrix} + \begin{pmatrix} 11 \\ 2 \\ a \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -10 \\ 1 + a \end{pmatrix}$$

[2 marks]

(v)

$$\overrightarrow{SR} = 1.5\,\overrightarrow{PQ}$$

$$\therefore\ a - 3 = 1.5\,(1 + a)$$

$$a - 3 = 1.5 + 1.5a$$

$$a = -9$$

[3 marks]

Answer 3

(i)

$$\cos^2\theta + \sin^2\theta = 1$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

[2 marks]

(ii)

$$2\cos^2\theta = 1 + \cos 2\theta$$

$$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

[2 marks]

(iii)

$$\int_0^{\frac{\pi}{2}} \cos^2\theta \ d\theta = \int_0^{\frac{\pi}{2}} \frac{1}{2} + \frac{1}{2}\cos 2\theta \ d\theta$$

$$= \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{4}$$

[3 marks]

Answer 4

$$x = -3\cot y$$

$$\frac{dx}{dy} = 3\csc^2 y$$

$$= 3\,(\cot^2 y + 1)$$

$$= 3\left(\left(-\frac{x}{3}\right)^2 + 1 \right)$$

$$= 3\left(\frac{x^2}{9} + \frac{9}{9} \right)$$

$$= \frac{3}{9}\left(x^2 + 9\right)$$

$$= \frac{x^2 + 9}{3}$$

$$\frac{dy}{dx} = \frac{3}{x^2 + 9}$$

Thus $a = 3$ and $b = 9$

[5 marks]