5.4 Answers

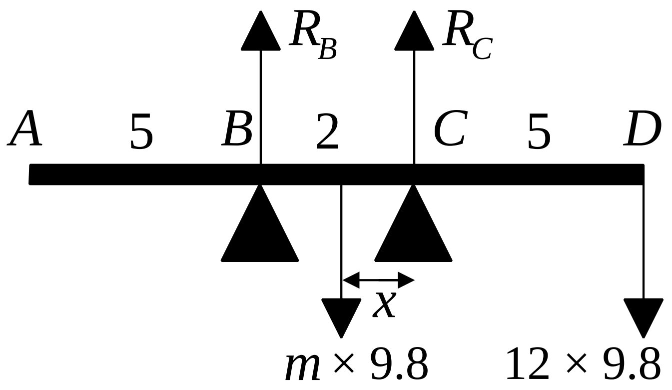
5.4.1 Solution to 5.2 Example

(i) By symmetry, if the mast could be modelled by a rod that was uniform, the number of bottles of water to be on the verge of tilting would be the same when they were at end *A* as at end *D*.

(ii) The centre of mass has to be between the supports as if it wasn't one end of the mast would be on the ground.

Let x be the distance of the centre of mass from support *C*.

When the 6×2 litre bottles, mass 12 kg, are suspended from *D*,

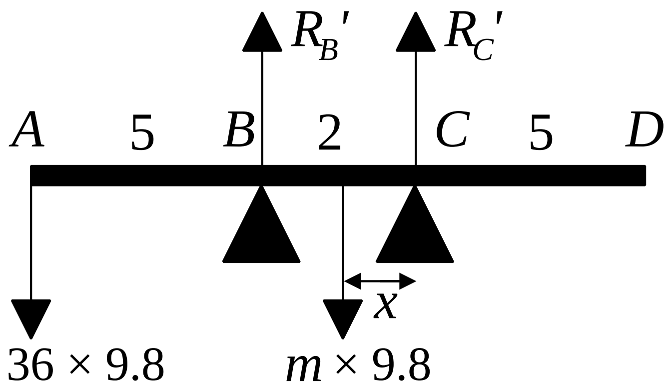


On the verge of tilting, $R_B = 0$ N. Taking moments about *C*,

$$m \times 9.8 \times x = 12 \times 9.8 \times 5$$

$$mx = 60$$

When the 18×2 litre bottles, mass 36 kg, are suspended from *A*,



On the verge of tilting, $R_C' = 0$ N. Taking moments about *B*,

$$m \times 9.8 \times (2 - x) = 36 \times 9.8 \times 5$$

$$2m - mx = 180$$

Combining equations: $\quad 2m - 60 = 180$

$$2m = 240$$

$$m = 120 \text{ kg}$$

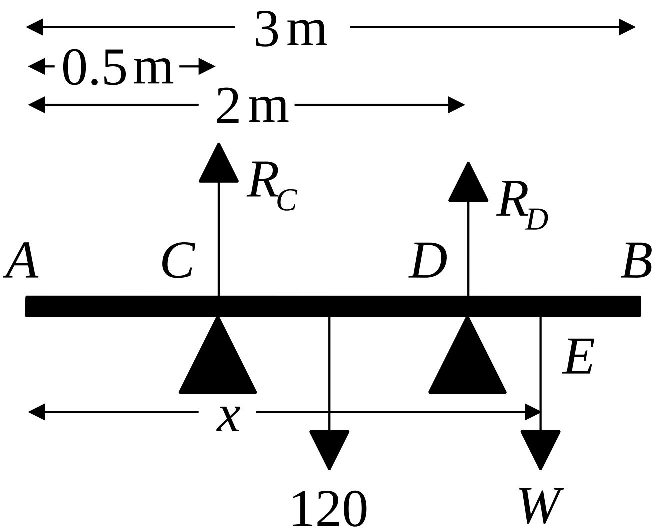
(iii) Substituting the value of m back into the equation $mx = 60$ gives, $x = 0.5$

Thus the centre of mass is 5.5 metres from end *D*

5.4.2 Solutions to 5.3 Exercise

Answer 1

(i)



Location of E not known

(ii)

Taking moments about A,

$$Wx + 120 \times 1.5 = R_C \times 0.5 + R_D \times 2$$
$$Wx + 180 = 2 \times R_D \times 0.5 + 2\,R_D$$
$$Wx + 180 = 3\,R_D$$

(iii)

Balancing forces vertically,

$$R_C + R_D = 120 + W$$
$$2 \times R_D + R_D = 120 + W$$
$$3\,R_D = 120 + W$$

(iv) Equating previous answers,

$$\therefore\ Wx + 180 = 120 + W$$
$$60 = W - Wx$$
$$60 = W\,(1 - x)$$
$$W = \frac{60}{1 - x} \qquad \square$$

(v) As a length cannot be negative, $x \geqslant 0$

A weight W also cannot be negative, so $x < 1$

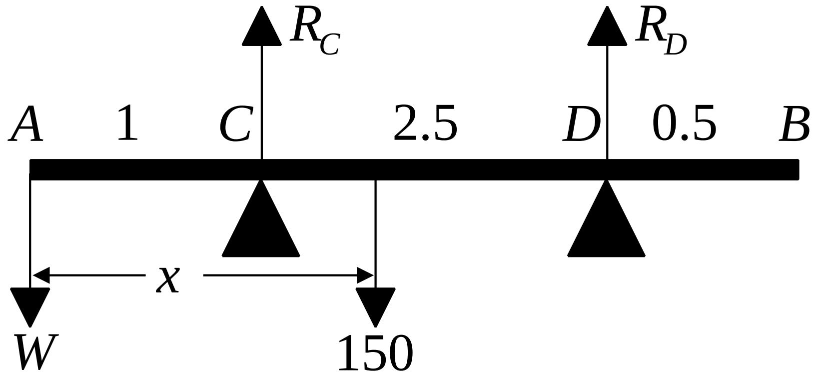
$\therefore\ 0 \leqslant x < 1$

The minimum weight that could be used is thus 60 N.

There is no upper limit, according to this model.

Answer 2

(i)



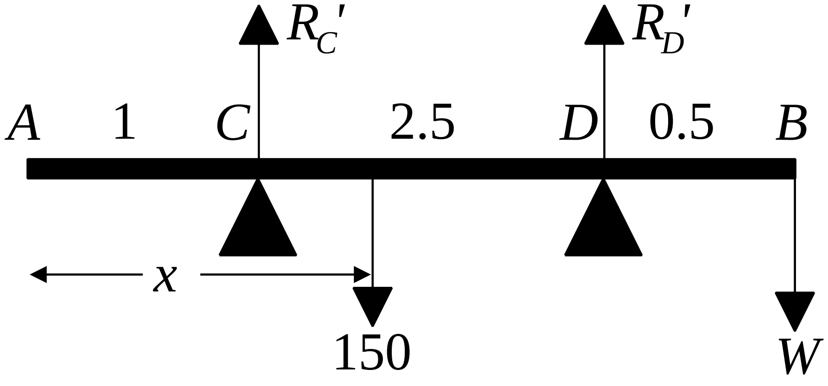
(ii) Balancing forces vertically,

$$R_C + R_D = W + 150$$
$$100 + R_D = W + 150$$
$$R_D = W + 50 \quad \square$$

(iii) Taking moments about A,

$$150x = R_C \times 1 + R_D \times 3.5$$
$$150x = 100 \times 1 + (W + 50) \times 3.5$$
$$150x = 100 + 3.5W + 175$$
$$150x = 275 + 3.5W$$
$$300x = 550 + 7W \quad \square$$

(iv) Let R_C' and R_D' be the different reactions at C and D.



Balancing forces vertically,

$$R_C' + R_D' = W + 150$$
$$52 + R_D' = W + 150$$
$$R_D' = W + 98 \quad \square$$

Taking moments about A,

$$150 \times x + W \times 4 = R_C' \times 1 + R_D' \times 3.5$$

$$150x + 4W = 52 \times 1 + (W + 98) \times 3.5$$

$$150x + 4W = 52 + 3.5W + 343$$

$$150x = 395 - 0.5W$$

$$300x = 790 - W$$

(v)

$$550 + 7W = 790 - W$$

$$8W = 240$$

$$W = 30 \text{ N}$$

Thus,

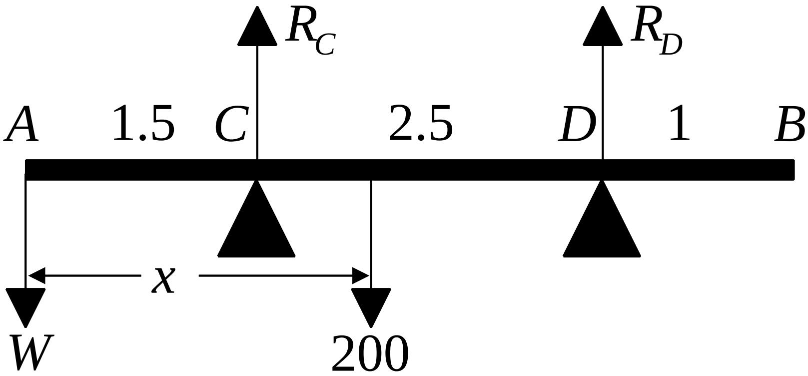
$$300x = 790 - W$$

$$300x = 790 - 30$$

$$x = 2.53 \text{ metres}$$

Answer 3

(a)



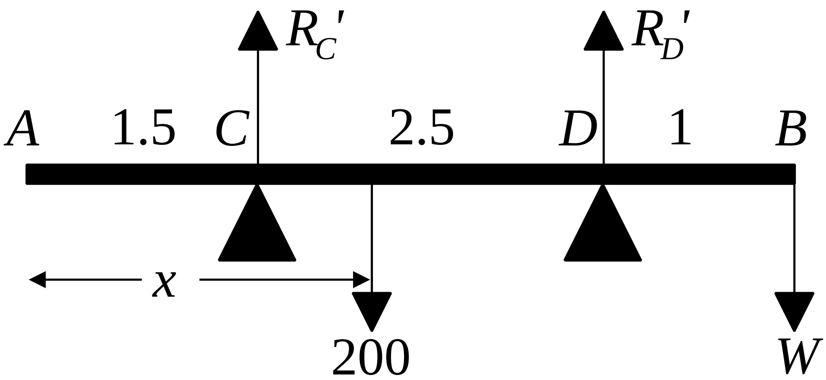
Balancing forces vertically,

$$\begin{aligned} R_C + R_D &= W + 200 \\ 160 + R_D &= W + 200 \\ R_D &= W + 40 \end{aligned}$$

Taking moments about A,

$$\begin{aligned} 200x &= R_C \times 1.5 + R_D \times 4 \\ 200x &= 160 \times 1.5 + (W + 40) \times 4 \\ 200x &= 240 + 4W + 160 \\ 200x &= 400 + 4W \\ 50x &= 100 + W \\ 50x - W &= 100 \qquad \square \end{aligned}$$

(b) Let R_C' and R_D' be the different reactions at C and D.



Balancing forces vertically,

$$\begin{aligned} R_C' + R_D' &= W + 200 \\ 50 + R_D' &= W + 200 \\ R_D' &= W + 150 \qquad \square \end{aligned}$$

Taking moments about A,

$$200 \times x + W \times 5 = R_C' \times 1.5 + R_D' \times 4$$

$$200x + 5W = 50 \times 1.5 + (W + 150) \times 4$$

$$200x + 5W = 75 + 4W + 600$$

$$200x + W = 675$$

Solving the two equations simultaneously,

$$50x - W = 100$$

$$200x + W = 675$$

ADD

$$250x = 775$$

$$x = 3.10 \text{ metres}$$

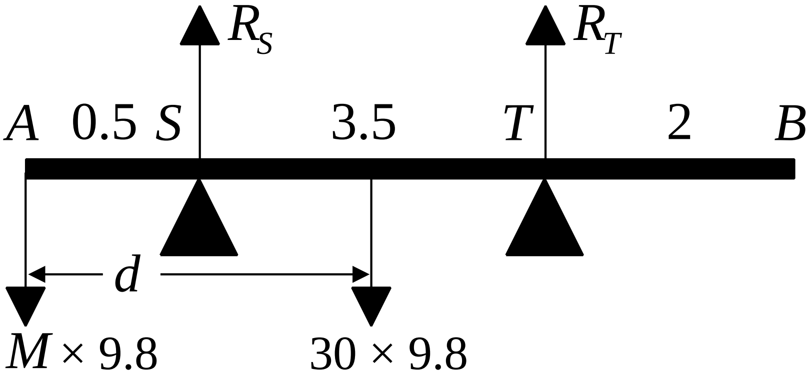
Thus,

$$200x + W = 675$$

$$W = 675 - 200 \times 3.10$$

$$= 55 \text{ N}$$

Answer 4



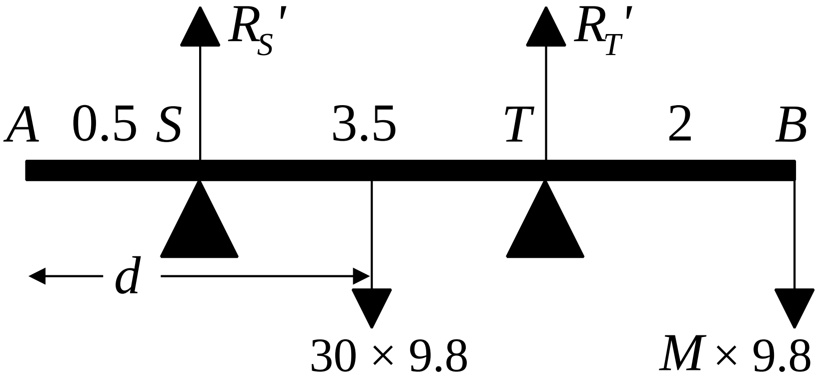
When about to tilt about S, $R_T = 0$ N

Taking moments about S,

$$30 \times 9.8 \times (d - 0.5) = M \times 9.8 \times 0.5$$

$$30d - 15 = 0.5M$$

$$M = 60d - 30$$



When about to tilt about T, $R_S' = 0$ N

Taking moments about T,

$$M \times 9.8 \times 2 = 30 \times 9.8 \times (4 - d)$$

$$2M = 120 - 30d$$

$$M = 60 - 15d$$

Combining the two equations,

$$60d - 30 = 60 - 15d$$

$$75d = 90$$

$$d = 1.2$$

$$\therefore\ M = 42 \text{ N}$$