Twenty-One Today
N U M B E R 1
Solutions

Answer 1

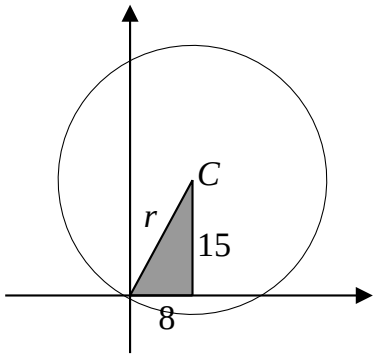
$$\frac{1}{\sqrt{7}-1} \times \frac{(\sqrt{7}+1)}{(\sqrt{7}+1)} = \frac{\sqrt{7}+1}{7-\sqrt{7}+\sqrt{7}-1}$$
$$= \frac{\sqrt{7}+1}{6}$$

Answer 2

By the factor theorem, $f(1) = 0$
So,

$$4 \times 1^3 - 7 \times 1^2 + a \times 1 + 13 = 0$$
$$4 - 7 + a + 13 = 0$$
$$a = -10$$

Answer 3



$$r^2 = 15^2 + 8^2$$
$$= 225 - 64$$
$$= 289$$
$$r = 17$$

Equation of the circle is,

$$(x-8)^2 + (y-15)^2 = 17^2$$

Answer 4

$$\log_m 45 - 2\log_m 3 = \log_m 45 - \log_m 3^2$$
$$= \log_m \left(\frac{45}{9}\right)$$
$$= \log_m 5$$

Answer 5

$$\log_m 45 - 2\log_m 3 = 1$$
$$\log_m 5 = 1$$
$$\therefore m^1 = 5 \quad \text{i.e. } m = 5$$

Answer 6

$$f(x) = 12x^{-2}$$
$$\int f(x)\,dx = \frac{12x^{-1}}{-1} + c$$
$$= -\frac{12}{x} + c$$

Answer 7

$$f'(x) = \left[x^2 + 10x\right] + 27$$
$$= \left[(x+5)^2 - 25\right] + 27$$
$$= (x+5)^2 + 2$$

The smallest that $(x+5)^2$ can be is zero.
Even then, 2 is added on.
$\therefore f'(x)$ is always positive.
i.e. $f(x)$ is an increasing function.

Answer 8
By integration,

$$y = x^2 + 5x + c$$
$$-1 = 3^2 + 5 \times 3 + c$$
$$c = -25$$
$$\therefore y = x^2 + 5x - 25$$

Answer 9
Lines (b) and (c) are mutually perpendicular because the product of their gradients is -1
That is,

$$\frac{1}{2} \times (-2) = -1$$

Answer 10

By the factor theorem,

$$f(1) = 0$$

$$p + q = -2$$

By the remainder theorem,

$$f(2) = -7$$

$$2p + q = -19$$

Solve these two equations simultaneously to obtain,

$$p = -17 \text{ and } q = 15$$

Answer 11

Method 1

$$7^x = 102$$

take the log_e of both sides

$$ln\, 7^x = ln\, 102$$

$$x\, ln\, 7 = ln\, 102$$

$$x = \frac{ln\, 102}{ln\, 7}$$

$$x = 2.38$$

Method 2

$$log_a b = c \quad \Leftrightarrow \quad a^c = b$$

The "jump out of logs" manoeuvre used backwards gives

$$7^x = 102 \Leftrightarrow log_7 102 = x$$

$$x = 2.38$$

Answer 12

$$\left(2^x\right)^2 - 5\left(2^x\right) + 6 = 0$$

$$\text{Let } z = 2^x$$

$$z^2 - 5z + 6 = 0$$

$$(z - 3)(z - 2) = 0$$

Either	or
$2^x = 3$	$2^x = 2$
$x = 1.58$	$x = 1$

Answer 13

Differentiation

Answer 14

$$\begin{aligned} Diameter &= \sqrt{(4-(-8))^2+(5-1)^2} \\ &= \sqrt{12^2+4^2} \\ &= \sqrt{160} \\ &= 4\sqrt{10} \\ \therefore\ Radius &= 2\sqrt{10} \\ Midpoint &= \left(\frac{-8+4}{2}, \frac{1+5}{2}\right) \\ &= (-2,\ 3) \end{aligned}$$

Equation of circle : $(x+2)^2+(y-3)^2 = 40$

Answer 15

Let $x = -1$, for example.

The statement given then becomes, $2 < 0$, which is not true.

Answer 16

$$2\cos^2\theta - 5\cos\theta + 2 = 0$$

$$(2\cos\theta - 1)(\cos\theta - 2) = 0$$

Either or

$\cos\theta = \frac{1}{2}$ (*two solutions*) $\cos\theta = 2$ (*no solutions*)

$$\theta = 60°,\ 300°$$

Answer 17

$$\begin{aligned} (2x+1)^5 &= 32 \\ 2x+1 &= 2 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

Answer 18

$$\begin{aligned}\left(\frac{x}{3}-1\right)^3 &= +1\times\left(\frac{x}{3}\right)^3\times(-1)^0\\ &\quad +3\times\left(\frac{x}{3}\right)^2\times(-1)^1\\ &\quad +3\times\left(\frac{x}{3}\right)^1\times(-1)^2\\ &\quad +1\times\left(\frac{x}{3}\right)^0\times(-1)^3\\ &= \frac{x^3}{27}-\frac{x^2}{3}+x-1\end{aligned}$$

Answer 19

Crosses x-axis at $(-5,\ 0)$ or $(5,\ 0)$

Crosses y-axis at $(0, 25)$

Answer 20

$x \leqslant -5$ or $x \geqslant 5$

Answer 21

By symmetry,

$$\begin{aligned}2\int_0^5 25-x^2\,dx &= 2\left[25x-\frac{x^3}{3}\right]_0^5\\ &= 2\left[25\times 5-\frac{5^3}{3}\right]\\ &= 166\frac{2}{3}\end{aligned}$$