

2.5 Answers

GCSE Mathematics The Classification of Numbers

2.5.1 Solution to 2.2 Terminating Decimal Example

$$0.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8} \text{ which is in the form of a rational number. } \square$$

[2 marks]

2.5.2 Solution to 2.3 Repeating Pattern Decimal Example

$$\begin{array}{r} 100x = 63.63\ 63\ 63\ 63\ \dots \\ \text{SUBTRACT} \quad x = 0.63\ 63\ 63\ 63\ \dots \\ \hline 99x = 63 \\ x = \frac{63}{99} \\ x = \frac{7}{11} \text{ which is in the form of a rational number. } \square \end{array}$$

[3 marks]

2.5.3 Solution to 2.4 Exercise

Answer 1

$$0.225 = \frac{225}{1000} = \frac{45}{200} = \frac{9}{40} \text{ which is in the form of a rational number. } \square$$

[2 marks]

Answer 2

$$\begin{array}{r} 100x = 36.36\ 36\ 36\ 36\ \dots \\ \text{SUBTRACT} \quad x = 0.36\ 36\ 36\ 36\ \dots \\ \hline 99x = 36 \\ x = \frac{36}{99} \\ x = \frac{4}{11} \text{ which is in the form of a rational number. } \square \end{array}$$

[3 marks]

Answer 3

$$100x = 75.75\ 75\ 75\ 75\ \dots$$

$$\text{SUBTRACT} \quad x = 0.75\ 75\ 75\ 75\ \dots$$

$$99x = 75$$

$$x = \frac{75}{99}$$

$$x = \frac{25}{33} \text{ which is in the form of a rational number. } \square$$

[3 marks]**Answer 4**

$$1000x = 351.351\ 351\ 351\ 351\ \dots$$

$$\text{SUBTRACT} \quad x = 0.351\ 351\ 351\ 351\ \dots$$

$$999x = 351$$

$$x = \frac{351}{999}$$

$$x = \frac{13}{37} \text{ which is in the form of a rational number. } \square$$

[3 marks]**Answer 5**

$$(i) \quad 0.875 = \frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$$

which is in the form $\frac{p}{q}$ for integer p and q with $q \neq 0$.

[2 marks]

$$(ii) \quad \square \quad 0.875 \text{ is a rational number, } \mathbb{Q}$$

[1 mark]**Answer 6**

$$2.52 = \frac{252}{100} = \frac{126}{50} = \frac{63}{25} \text{ which is in the form } \frac{p}{q} \text{ for integer } p \text{ and } q \text{ with } q \neq 0.$$

[2 marks]**Answer 7**

$$(i) \quad 23 = \frac{23}{1} \text{ which is in the form } \frac{p}{q} \text{ for integer } p \text{ and } q \text{ with } q \neq 0.$$

[1 mark]

$$(ii) \quad \text{For any integer } n, \quad n = \frac{n}{1} \text{ which is in the form } \frac{p}{q} \text{ for integer } p$$

and q with $q \neq 0$ which is the form of a rational number.

$\therefore$ All integers are rational. $\square$

[2 marks]

Answer 8

$$100000x = 14634.14634\ 14634\ 14634\ 14634\ \dots$$

$$\text{SUBTRACT} \quad x = 0.14634\ 14634\ 14634\ 14634\ \dots$$

$$99999x = 14634$$

$$x = \frac{14634}{99999}$$

$$x = \frac{6}{41} \text{ which is in the form of a rational number. } \square$$

[5 marks]**Answer 9**

$$(i) \quad 10x = 4.4\ 4\ 4\ 4\ 4\ 4\ \dots$$

$$\text{SUBTRACT} \quad x = 0.4\ 4\ 4\ 4\ 4\ 4\ \dots$$

$$9x = 4$$

$$x = \frac{4}{9} \text{ which is in the form of a rational number. } \square$$

[2 marks]

$$(ii) \quad 10x = a.a\ a\ a\ a\ a\ a\ \dots$$

$$\text{SUBTRACT} \quad x = 0.a\ a\ a\ a\ a\ a\ \dots$$

$$9x = a$$

$$x = \frac{a}{9} \text{ which is in the form of a rational number. } \square$$

[5 marks]**Answer 10**

$$(i) \quad 100x = 12.2\ 2\ 2\ 2\ 2\ 2\ \dots$$

$$\text{SUBTRACT} \quad 10x = 1.2\ 2\ 2\ 2\ 2\ 2\ \dots$$

$$90x = 11$$

$$x = \frac{11}{90} \text{ which is in the form of a rational number. } \square$$

[2 marks]

$$(ii) \quad 100x = 1a.a\ a\ a\ a\ a\ a\ \dots$$

$$\text{SUBTRACT} \quad 10x = 0.a\ a\ a\ a\ a\ a\ \dots$$

$$90x = 10 + a$$

$$x = \frac{10 + a}{90} \text{ which is in the form of a rational number. } \square$$

[5 marks]

Answer 11

$$100x = ab.ab\ ab\ ab\ ab\ \dots$$

$$\text{SUBTRACT} \quad x = 0.ab\ ab\ ab\ ab\ \dots$$

$$99x = ab$$

$$x = \frac{10a + b}{99} \text{ which is in the form of a rational number. } \square$$

[5 marks]**Answer 12**

$$0.\dot{0}2197\dot{8}$$

$$1000000x = 021978.021978\ 021978\ 021978\ \dots$$

$$\text{SUBTRACT} \quad x = 0.021978\ 021978\ 021978\ \dots$$

$$999999x = 021978$$

$$x = \frac{21978}{999999}$$

$$x = \frac{2}{91} \text{ which is in the form of a rational number. } \square$$

[4 marks]**Answer 13**

Take the obvious equivalence $0.33333333\ldots = \frac{1}{3}$

and multiply both sides by 3 $0.99999999\ldots = 1 \quad \square$

[3 marks]