

4.4 Answers

A-Level Pure Mathematics, Year 2 Geometric Progressions

4.4.1 Solution to 4.2 Example

The problem is that to use the formula to work out the sum of the series, the number of terms, n , needs to be found first.

$$a r^{n-1} = 49152$$

$$3 \times 2^{n-1} = 49152$$

$$2^{n-1} = 16384$$

take the natural log of both sides

$$\ln 2^{n-1} = \ln 16384$$

$$(n - 1) \ln 2 = \ln 16384$$

$$n - 1 = \frac{\ln 16384}{\ln 2}$$

$$n - 1 = 14$$

$$n = 15$$

$$\begin{aligned} S_{15} &= \frac{3(2^{15} - 1)}{(2 - 1)} \\ &= 98301 \end{aligned}$$

[4 marks]

4.4.2 Solutions to 4.3 Exercise

Answer 1

$$ar^{n-1} > 1\,000\,000$$

$$2 \times 3^{n-1} > 1\,000\,000$$

$$3^{n-1} > 500\,000$$

take the natural log of both sides

$$\ln 3^{n-1} > \ln 500\,000$$

$$(n-1)\ln 3 > \ln 500\,000$$

$$n-1 > \frac{\ln 500\,000}{\ln 3}$$

$$n-1 > 11.94$$

$$n > 12.94$$

$$\text{i.e. } n = 13$$

The 13th term is the first to exceed one million

2, 6, 18, 54, 162, 486, 1458, 4374, 13122, 39366, 118098, 354294, 1062882 ...

[4 marks]

Answer 2

The 17th term

[4 marks]

Answer 3

$n = 10$, Sum = 29524

[4 marks]

Answer 4

$n > 11.7 \therefore$ First exceeds 1000 in the 12th year of 2023 (Not 11th !)

$n > 11.7 \therefore$ First exceeds 1000 in the 12th year of 2023 (Not the 11th !)

[4 marks]

Answer 5

$n = 31$, Sum = 715 827 883

[4 marks]

Answer 6

(a) Told $ar = 4$ and $\frac{a}{1-r} = 25$

$$a = 25(1-r)$$

$$\frac{4}{r} = 25(1-r)$$

$$4 = 25r - 25r^2$$

$$25r^2 - 25r + 4 = 0 \quad \square$$

(b) 0.2 or 0.8

(c) 20 or 5

(d) $S_n = \frac{a(1-r^n)}{(1-r)}$ but $\frac{a}{1-r} = 25$

so $S_n = 25(1-r^n) \quad \square$

(e) $n > \frac{\log 0.04}{\log 0.8} \quad n = 15$

Note : Division of an inequality by a negative quantity reverses the inequality.
 $\ln x$ is negative for $0 < x < 1$

[4, 2, 2, 1, 2 marks]

Answer 7

(a) 0.072

(b) 25

(c) $\frac{5(1-0.8^k)}{1-0.8} > 24.95$

$$k > \frac{\log 0.002}{\log 0.8}$$

Note : Division of an inequality by a negative quantity reverses the inequality.
 $\ln x$ is negative for $0 < x < 1$

(d) 28

[2, 2, 4, 1]