

1.3 Answers

A-Level Pure Mathematics : Year 1 Algebra of Surds and Indices II

1.3.1 Solutions to Introductory Challenges

The key idea is to organise the multiplication process in some way.

I suggest a table.

Calculators will not do the whole calculation exactly, but will do the individual entries in the following tables exactly.

The individual entries can be done as mental arithmetic.

Challenge #1

	2	$7\sqrt{2}$	$-5\sqrt{3}$
4	8	$28\sqrt{2}$	$-20\sqrt{3}$
$3\sqrt{2}$	$6\sqrt{2}$	42	$-15\sqrt{6}$
$2\sqrt{3}$	$4\sqrt{3}$	$14\sqrt{6}$	-30

$$\begin{aligned} & (2 + 7\sqrt{2} - 5\sqrt{3})(4 + 3\sqrt{2} + 2\sqrt{3}) \\ &= 20 + 34\sqrt{2} - 16\sqrt{3} - \sqrt{6} \end{aligned}$$

[6 marks]

Challenge #2

It will help to think of $\sqrt{6}$ as $\sqrt{2} \times \sqrt{3}$ if doing this by mental arithmetic.

	$3\sqrt{2}$	$4\sqrt{3}$	$7\sqrt{6}$
$5\sqrt{2}$	30	$20\sqrt{6}$	$70\sqrt{3}$
$-2\sqrt{3}$	$-6\sqrt{6}$	-24	$-42\sqrt{2}$
$3\sqrt{6}$	$18\sqrt{3}$	$36\sqrt{2}$	126

$$\begin{aligned} & (3\sqrt{2} + 4\sqrt{3} + 7\sqrt{6})(5\sqrt{2} - 2\sqrt{3} + 3\sqrt{6}) \\ &= 132 - 6\sqrt{2} + 88\sqrt{3} + 14\sqrt{6} \end{aligned}$$

[6 marks]

1.3.2 Solutions to 1.2 Exercise

Answer 1

$$\begin{aligned} \text{(i)} \quad (5 + \sqrt{7})^2 &= 25 + 10\sqrt{7} + 7 \\ &= 32 + 10\sqrt{7} \end{aligned}$$

[2 marks]

$$\begin{aligned} \text{(ii)} \quad (2 + 5\sqrt{3})(4 - 2\sqrt{3}) &= 8 - 4\sqrt{3} + 20\sqrt{3} - 30 \\ &= -22 + 16\sqrt{3} \end{aligned}$$

[3 marks]

$$\begin{aligned} \text{(iii)} \quad &\text{“difference of two squares”} \\ (3 + \sqrt{2})(3 - \sqrt{2})(2 + \sqrt{5}) &= (7)(2 + \sqrt{5}) \\ &= 14 + 7\sqrt{5} \end{aligned}$$

[4 marks]

$$\begin{aligned} \text{(iv)} \quad (6 + 2\sqrt{3} - \sqrt{5})(7 + \sqrt{3}) \\ &= 42 + 6\sqrt{3} + 14\sqrt{3} + 6 - 7\sqrt{5} - \sqrt{15} \\ &= 48 + 20\sqrt{3} - 7\sqrt{5} - \sqrt{15} \end{aligned}$$

[5 marks]

Answer 2

(i) Integers are;

$$\textcircled{6} \quad \pi \quad \sqrt{8} \quad \textcircled{-4} \quad \textcircled{\sqrt{25}} \quad 0.125 \quad \frac{4}{5} \quad \textcircled{\frac{81}{3}}$$

[1 mark]

(ii) Square-free are;

$$\textcircled{7} \quad 121 \quad 50 \quad 12 \quad \textcircled{6} \quad 99 \quad \textcircled{65} \quad 147$$

[2 marks]

Answer 3

$$\text{(i)} \quad \sqrt{8} = 2\sqrt{2}$$

[1 mark]

$$\text{(ii)} \quad 5\sqrt{20} = 10\sqrt{5}$$

[1 mark]

$$\begin{aligned} \text{(iii)} \quad (3\sqrt{2})^3 - 4\sqrt{2} &= 27 \times 2\sqrt{2} - 4\sqrt{2} \\ &= 50\sqrt{2} \end{aligned}$$

[2 marks]

$$\begin{aligned} \text{(iv)} \quad \sqrt{12} + 3\sqrt{48} + \sqrt{75} &= 2\sqrt{3} + 12\sqrt{3} + 5\sqrt{3} \\ &= 19\sqrt{3} \end{aligned}$$

[2 marks]

Answer 4

$$\begin{aligned} & (1 + \sqrt{5}) (1 - \sqrt{5})^3 \\ &= (1 + \sqrt{5}) (1 - \sqrt{5}) (1 - \sqrt{5})^2 \\ &= (-4) (1 - \sqrt{5})^2 \\ &= (-4) (1 - 2\sqrt{5} + 5) \\ &= (-4) (6 - 2\sqrt{5}) \\ &= -24 + 8\sqrt{5} \end{aligned}$$

[3 marks]

Answer 5

	5	$3\sqrt{2}$	$6\sqrt{3}$
8	40	$24\sqrt{2}$	$48\sqrt{3}$
$4\sqrt{2}$	$20\sqrt{2}$	24	$24\sqrt{6}$
$-2\sqrt{3}$	$-10\sqrt{3}$	$-6\sqrt{6}$	-36

$$\begin{aligned} & (5 + 3\sqrt{2} + 6\sqrt{3}) (8 + 4\sqrt{2} - 2\sqrt{3}) \\ &= 28 + 44\sqrt{2} + 38\sqrt{3} + 18\sqrt{6} \end{aligned}$$

[6 marks]

Answer 6

Only (i) is square-free

[2 marks]

Answer 7

$$\begin{aligned} \text{LHS} &= \sqrt{2 \times 3^3 \times 5^2} \\ &= \sqrt{3^2 \times 5^2 \times 2 \times 3} \\ &= \sqrt{(3 \times 5)^2 \times (2 \times 3)} \\ &= \sqrt{15^2} \times \sqrt{6} \\ &= 15\sqrt{6} \\ &= \text{RHS} \quad \square \end{aligned}$$

[2 marks]

Answer 8

$$\begin{aligned} \sqrt{a^3 \times b^5} &= \sqrt{a^2 \times b^4 \times a \times b} \\ &= a b^2 \sqrt{ab} \\ \text{i.e. } x &= a b^2 \quad \text{and} \quad y = ab \end{aligned}$$

[2 marks]

Answer 9

$$\begin{aligned}\sqrt{5^{52} \times 7 \times 13^{95}} &= \sqrt{5^{52} \times 13^{94} \times 7 \times 13} \\ &= 5^{26} \times 13^{47} \times \sqrt{7 \times 13}\end{aligned}$$

[2 marks]**Answer 10**

It will help to think of $\sqrt{10}$ as $\sqrt{2} \times \sqrt{5}$ if doing this by mental arithmetic.

	$7\sqrt{2}$	$-3\sqrt{5}$	$4\sqrt{10}$
$9\sqrt{2}$	126	$-27\sqrt{10}$	$72\sqrt{5}$
$2\sqrt{5}$	$14\sqrt{10}$	-30	$40\sqrt{2}$
$2\sqrt{10}$	$28\sqrt{5}$	$-30\sqrt{2}$	80

$$\begin{aligned}& (7\sqrt{2} - 3\sqrt{5} + 4\sqrt{10}) (9\sqrt{2} + 2\sqrt{5} + 2\sqrt{10}) \\ &= 176 + 10\sqrt{2} + 100\sqrt{5} - 13\sqrt{10}\end{aligned}$$

[6 marks]**Answer 11**

A 'by hand' division ladder...

$$\begin{array}{r} 2 \overline{) 1728} \\ 2 \overline{) 864} \\ 2 \overline{) 432} \\ 2 \overline{) 216} \\ 2 \overline{) 108} \\ 2 \overline{) 54} \\ 3 \overline{) 27} \\ 3 \overline{) 9} \\ 3 \overline{) 3} \\ \underline{\quad} 1 \end{array}$$

... in order to deduce (without using a calculator's FACT button) that...

$$\begin{aligned}1728 &= 2^6 \times 3^3 \\ \therefore \sqrt[3]{1728} &= 12\end{aligned}$$

[4 marks]