

2.5 Answers

A-Level Pure Mathematics : Year 1 Algebra of Surds and Indices II

2.5.1 Solutions to 2.2 Example (GCSE Level 8 or 9)

- (i) No direct penalty for not knowing that $a \in \mathbb{Z}$ means that a is an integer.
1 mark for stating $a = 0$ and $a = 1$ are excluded to avoid division by zero.
1 mark for stating that division by zero is undefined in mathematics.
Not both of $a = 0$ and $a = 1$ must be mentioned, either one will suffice.
1 mark for explaining that all negative numbers have been excluded to avoid taking the square root of a negative number which would give a non-real (i.e. complex number) value to the function. Any answer that captures an aspect of this gets the mark which is more for realising that there are two separate reasons behind restricting the domain, one to do with division by zero and the other for the taking of negative square roots.

[3 marks]

(ii)
$$\begin{aligned}\frac{a}{a - \sqrt{a}} &= \frac{a}{a - \sqrt{a}} \times \frac{a + \sqrt{a}}{a + \sqrt{a}} && \text{Key step !} \\ &= \frac{a(a + \sqrt{a})}{a^2 - a} \\ &= \frac{a(a + \sqrt{a})}{a(a - 1)} \\ &= \frac{a + \sqrt{a}}{a - 1}\end{aligned}$$

[3 marks]

2.5.2 Solution to 2.3 Example (A-Level)

- (i) Rationals can be written $\frac{p}{q}$ for integer values of p and q and $q \neq 0$

[2 marks]

(ii)
$$\begin{aligned}\text{LHS} &= \frac{\sqrt{5} - \sqrt{13}}{3\sqrt{5} + \sqrt{13}} \\ &= \frac{\sqrt{5} - \sqrt{13}}{3\sqrt{5} + \sqrt{13}} \times \frac{3\sqrt{5} - \sqrt{13}}{3\sqrt{5} - \sqrt{13}} \\ &= \frac{15 - \sqrt{65} - 3\sqrt{65} + 13}{45 - 13} \\ &= \frac{28 - 4\sqrt{65}}{32} \\ &= \frac{7}{8} - \frac{1}{8}\sqrt{65} \\ &= \text{RHS} \quad \square\end{aligned}$$

[4 marks]

2.5.3 Solutions to 2.4 Exercise

Answer 1

$$\begin{aligned}
 \text{LHS} &= \frac{3\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}} \\
 &= \frac{3\sqrt{7} - \sqrt{5}}{\sqrt{7} + \sqrt{5}} \times \frac{\sqrt{7} - \sqrt{5}}{\sqrt{7} - \sqrt{5}} \\
 &= \frac{21 - 3\sqrt{35} - \sqrt{35} + 5}{7 - 5} \\
 &= \frac{26 - 4\sqrt{35}}{2} \\
 &= 13 - 2\sqrt{35} \\
 &= \text{RHS} \quad \square
 \end{aligned}$$

[4 marks]

Answer 2

Method 1

$$\begin{aligned}
 \text{LHS} &= \frac{1}{(2 + \sqrt{3})^2} \\
 &= \frac{1}{4 + 4\sqrt{3} + 3} \\
 &= \frac{1}{7 + 4\sqrt{3}} \times \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}} \\
 &= \frac{7 - 4\sqrt{3}}{49 - 48} \\
 &= 7 - 4\sqrt{3} \\
 &= \text{RHS} \quad \square
 \end{aligned}$$

Method 2 (Clever)

$$\begin{aligned}
 \text{LHS} &= \frac{1}{(2 + \sqrt{3})^2} \\
 &= \frac{1}{(2 + \sqrt{3})^2} \times \frac{(2 - \sqrt{3})^2}{(2 - \sqrt{3})^2} \\
 &= \frac{(2 - \sqrt{3})^2}{(4 - 3)(4 - 3)} \\
 &= \frac{4 - 4\sqrt{3} + 3}{1} \\
 &= 7 - 4\sqrt{3} \\
 &= \text{RHS} \quad \square
 \end{aligned}$$

[4 marks]

Answer 3

$$\begin{aligned}
 \frac{5}{\sqrt{75} - \sqrt{50}} &= \frac{5}{5\sqrt{3} - 5\sqrt{2}} \\
 &= \frac{1}{\sqrt{3} - \sqrt{2}} \times \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} \\
 &= \frac{\sqrt{3} + \sqrt{2}}{3 - 2} \\
 &= \sqrt{3} + \sqrt{2}
 \end{aligned}$$

...which is of the required form

[4 marks]

Answer 4

$$\begin{aligned}
 \text{(i)} \quad \text{Area } \Delta &= \frac{1}{2} \times \text{base} \times \text{height} \\
 9 &= \frac{1}{2} \times (5 + \sqrt{7}) \times \text{height} \\
 \text{height} &= \frac{18}{5 + \sqrt{7}} \\
 &= \frac{18}{5 + \sqrt{7}} \times \frac{5 - \sqrt{7}}{5 - \sqrt{7}} \\
 &= \frac{18(5 - \sqrt{7})}{25 - 7} \\
 &= \frac{18(5 - \sqrt{7})}{18} \\
 &= 5 - \sqrt{7}
 \end{aligned}$$

[3 marks]

$$\begin{aligned}
 \text{(ii)} \quad \text{Hypotenuse}^2 &= (5 + \sqrt{7})^2 + (5 - \sqrt{7})^2 \\
 &= 25 + 10\sqrt{7} + 7 + 25 - 10\sqrt{7} + 7 \\
 &= 64 \\
 \text{Hypotenuse} &= 8
 \end{aligned}$$

$$\begin{aligned}
 \text{Perimeter} &= 8 + 5 + \sqrt{7} + 5 - \sqrt{7} \\
 &= 18 \text{ cm}
 \end{aligned}$$

[3 marks]**Answer 5**

$$\begin{aligned}
 \text{LHS} &= \frac{3}{\sqrt{2}} + \frac{5}{\sqrt{3}} \\
 &= \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \frac{3\sqrt{2}}{2} + \frac{5\sqrt{3}}{3} \\
 &= \frac{3\sqrt{2}}{2} \times \frac{3}{3} + \frac{5\sqrt{3}}{3} \times \frac{2}{2} \\
 &= \frac{9\sqrt{2}}{6} + \frac{10\sqrt{3}}{6} \\
 &= \frac{9\sqrt{2} + 10\sqrt{3}}{6} \\
 &= \text{RHS} \quad \square
 \end{aligned}$$

[4 marks]

Answer 6

$$\begin{aligned}\frac{1}{\sqrt{a}} + \frac{4\sqrt{a}}{a} &= \frac{1}{\sqrt{a}} \times \frac{\sqrt{a}}{\sqrt{a}} + \frac{4\sqrt{a}}{a} \\ &= \frac{\sqrt{a} + 4\sqrt{a}}{a} \\ &= \frac{5\sqrt{a}}{a}\end{aligned}$$

[5 marks]

Answer 7

$$8 - x\sqrt{12} = \frac{8x}{\sqrt{3}}$$

Multiply through by $\sqrt{3}$

$$8\sqrt{3} - x\sqrt{3} \times \sqrt{12} = 8x$$

$$8\sqrt{3} - x\sqrt{36} = 8x$$

$$8\sqrt{3} - 6x = 8x$$

$$14x = 8\sqrt{3}$$

$$x = \frac{4\sqrt{3}}{7}$$

[5 marks]

Answers 8

$$\begin{aligned}\frac{p+q}{p-q} &= \frac{3-2\sqrt{2}+2-\sqrt{2}}{3-2\sqrt{2}-2+\sqrt{2}} \\ &= \frac{5-3\sqrt{2}}{1-\sqrt{2}} \\ &= \frac{5-3\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} \\ &= \frac{5+5\sqrt{2}-3\sqrt{2}-6}{1-2} \\ &= \frac{-1+2\sqrt{2}}{-1} \\ &= 1-2\sqrt{2}\end{aligned}$$

i.e. $m = 1$ and $n = -2$

[5 marks]

Answer 9

The initial step is to rationalise the denominators.

The clever way to do this is to note that they are all of the form,

$$\frac{1}{\sqrt{n} + \sqrt{n+1}}$$

So,

$$\begin{aligned} \frac{1}{\sqrt{n} + \sqrt{n+1}} &= \frac{1}{\sqrt{n} + \sqrt{n+1}} \times \frac{\sqrt{n} - \sqrt{n+1}}{\sqrt{n} - \sqrt{n+1}} \\ &= \frac{\sqrt{n} - \sqrt{n+1}}{n - (n+1)} \\ &= \frac{\sqrt{n} - \sqrt{n+1}}{-1} \\ &= \sqrt{n+1} - \sqrt{n} \end{aligned}$$

Now to make use of this in the given expression,

$$\begin{aligned} &\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{2021} + \sqrt{2022}} \\ &= \sqrt{2} - \sqrt{1} + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \dots + \sqrt{2022} - \sqrt{2021} \\ &= \sqrt{2022} - \sqrt{1} \\ &= \sqrt{2022} - 1 \end{aligned}$$

[7 marks]

Answer 10

$$\begin{aligned} \frac{7}{\sqrt{ab}} + \frac{b}{\sqrt{a}} + \frac{a}{\sqrt{b}} &= \frac{7}{\sqrt{ab}} + \frac{b}{\sqrt{a}} \times \frac{\sqrt{b}}{\sqrt{b}} + \frac{a}{\sqrt{b}} \times \frac{\sqrt{a}}{\sqrt{a}} \\ &= \frac{7 + b\sqrt{b} + a\sqrt{a}}{\sqrt{ab}} \\ &= \frac{7 + b\sqrt{b} + a\sqrt{a}}{\sqrt{ab}} \times \frac{\sqrt{ab}}{\sqrt{ab}} \\ &= \frac{7\sqrt{ab} + b^2\sqrt{a} + a^2\sqrt{b}}{ab} \end{aligned}$$

[6 marks]