3.3 Answers

3.3.1 Solutions to 3.1 Introductory Examples

Example #1 Answer

$$81\sqrt{3} = 3^a$$
$$3^4 \times 3^{\frac{1}{2}} = 3^a$$
$$3^{4.5} = 3^a$$
$$\therefore\ a = 4.5$$

[2 marks]

Example #2 Answer

$$(27x^{12})^{\frac{5}{3}} = 3^a x^b$$
$$(3x^4)^5 = 3^a x^b$$
$$3^5 x^{20} = 3^a x^b$$
$$\therefore\ a = 5 \text{ and } b = 20$$

[2 marks]

Example #3 Answer

$$(3 + \sqrt{c})(2\sqrt{c} - 3) = 1 + k\sqrt{c}$$
$$6\sqrt{c} - 9 + 2c - 3\sqrt{c} = 1 + k\sqrt{c}$$
$$(2c - 9) + 3\sqrt{c} = 1 + k\sqrt{c}$$
$$\therefore\ k = 3 \text{ and } 2c - 9 = 1$$
$$2c = 10$$
$$c = 5$$

[3 marks]

Example #4 Answer

$$\sqrt{10 + 2\sqrt{21}} = \sqrt{10 + 2\sqrt{3}\sqrt{7}}$$
$$= \sqrt{3 + 2\sqrt{3}\sqrt{7} + 7}$$
$$= \sqrt{(\sqrt{3})^2 + 2\sqrt{3}\sqrt{7} + (\sqrt{7})^2}$$

Recall that : $a^2 + 2ab + b^2 = (a + b)^2$

$$= \sqrt{(\sqrt{3} + \sqrt{7})^2}$$
$$= \sqrt{3} + \sqrt{7}$$
$$\therefore\ a = 3 \text{ and } b = 7 \qquad \text{as told } a < b$$

[3 marks]

3.3.2 Solutions (3.2 Exercise)

Answer 1

(i) 5 **(ii)** 3 **(iii)** $\frac{1}{9}$ **(iv)** 8 **(v)** $\frac{1}{3}$

(vi) $\frac{1}{1000}$ **(vii)** 3 **(viii)** 27 **(ix)** $\frac{1}{9}$ **(x)** 1

[5 marks]

Answer 2

$$\begin{aligned}
\sqrt{y} &= \sqrt{2^5 \times 3^4 \times 5^3} \\
&= \sqrt{2^4 \times 3^4 \times 5^2} \times \sqrt{2 \times 5} \\
&= 2^2 \times 3^2 \times 5 \times \sqrt{10} \\
&= 4 \times 9 \times 5 \times \sqrt{10} \\
&= 180\sqrt{10}
\end{aligned}$$

[2 marks]

Answer 3

(i) $\frac{16}{49}$ **(ii)** $\frac{4}{9}$ **(iii)** 1 **(iv)** $\frac{8}{3}$ **(v)** $\frac{5}{4}$

[5 marks]

Answer 4

(i) $-\frac{3}{2}$ **(ii)** $\frac{27\sqrt{2}}{32}$ **(iii)** $\frac{5\sqrt{2}}{4}$ **(iv)** $\frac{14 - 2\sqrt{5}}{11}$

[4 marks]

Answer 5

$$\begin{aligned}
8\sqrt{2} &= 2^a \\
2^3 \times 2^{\frac{1}{2}} &= 2^a \\
2^{3.5} &= 2^a \Rightarrow a = 3.5
\end{aligned}$$

[2 marks]

Answer 6

$$\begin{aligned}
\sqrt{3 + 2\sqrt{2}} &= \sqrt{2 + 2\sqrt{2}\sqrt{1} + 1} \\
&= \sqrt{(\sqrt{2})^2 + 2\sqrt{2}\sqrt{1} + (\sqrt{1})^2}
\end{aligned}$$

Recall that : $a^2 + 2ab + b^2 = (a + b)^2$

$$\begin{aligned}
&= \sqrt{(\sqrt{2} + \sqrt{1})^2} \\
&= \sqrt{2} + \sqrt{1}
\end{aligned}$$

$\therefore\ a = 2$ and $b = 1$ as told $a > b$

[3 marks]

Answer 7

$$\left(7 - \sqrt{c}\right)\left(4 + 2\sqrt{c}\right) = 6 + k\sqrt{c}$$
$$28 + 14\sqrt{c} - 4\sqrt{c} - 2c = 6 + k\sqrt{c}$$
$$\left(28 - 2c\right) + 10\sqrt{c} = 6 + k\sqrt{c}$$
$$\therefore\ k = 10 \text{ and } 28 - 2c = 6$$
$$2c = 22$$
$$c = 11$$

[3 marks]

Answer 8

$$\frac{1}{\sqrt[3]{9^4}} = 3^n$$
$$\left(9^4\right)^{-\frac{1}{3}} = 3^n$$
$$\left(3^8\right)^{-\frac{1}{3}} = 3^n$$
$$3^{-\frac{8}{3}} = 3^n$$
$$n = -\frac{8}{3}$$

[2 marks]

Answer 9

$$\left(a + \sqrt{5}\right)\left(3 + 2\sqrt{5}\right) = 31 + b\sqrt{5}$$
$$3a + 3\sqrt{5} + 2a\sqrt{5} + 10 = 31 + b\sqrt{5}$$
$$3a + 10 + (3 + 2a)\sqrt{5} = 31 + b\sqrt{5}$$
$$\therefore\ 3a + 10 = 31 \qquad \text{and} \qquad 3 + 2a = b$$
$$3a = 21$$
$$a = 7 \qquad \text{and so } b = 17$$

[3 marks]

Answer 10

Method 1

$$x = \sqrt{6 + 2\sqrt{5}} - \sqrt{6 - 2\sqrt{5}}$$
$$= \sqrt{5 + 2\sqrt{5}\sqrt{1} + 1} - \sqrt{5 - 2\sqrt{5}\sqrt{1} + 1}$$
$$= \sqrt{\left(\sqrt{5}\right)^2 + 2\sqrt{5}\sqrt{1} + \left(\sqrt{1}\right)^2} - \sqrt{\left(\sqrt{5}\right)^2 - 2\sqrt{5}\sqrt{1} + \left(\sqrt{1}\right)^2}$$
$$= \sqrt{\left(\sqrt{5} + \sqrt{1}\right)^2} - \sqrt{\left(\sqrt{5} - \sqrt{1}\right)^2}$$
$$= \sqrt{5} + \sqrt{1} - \sqrt{5} + \sqrt{1}$$
$$\therefore\ x = 2 \qquad \square$$

Method 2

$$
\begin{aligned}
x &= \sqrt{6 + 2\sqrt{5}} - \sqrt{6 - 2\sqrt{5}} \\
x^2 &= 6 + 2\sqrt{5} - 2\sqrt{6 + 2\sqrt{5}} \times \sqrt{6 - 2\sqrt{5}} + 6 - 2\sqrt{5} \\
&= 12 - 2\sqrt{(6 + 2\sqrt{5})(6 - 2\sqrt{5})} \\
&= 12 - 2\sqrt{36 + 12\sqrt{5} - 12\sqrt{5} - 20} \\
&= 12 - 2\sqrt{36 - 20} \\
&= 12 - 2\sqrt{16} \\
&= 4 \\
\therefore\ x &= 2 \qquad \square
\end{aligned}
$$

[4 marks]

Answer 11

$$
\begin{aligned}
p^m &= \frac{1}{p \times \sqrt[3]{p^2}} \\
&= \frac{1}{p} \times \frac{1}{\sqrt[3]{p^2}} \\
&= p^{-1} \times p^{-\frac{2}{3}} \\
&= p^{-\frac{5}{3}} \\
\therefore\ m &= -\frac{5}{3}
\end{aligned}
$$

[2 marks]

Answer 12

(a) $gh = a^4 \times b^2 \times c^5$

[2 marks]

(b) $\dfrac{g}{h} = a^2 \times b^0 \times c^{-1}$

$x = 2, \qquad y = 0, \qquad z = -1$

[3 marks]

Answer 13

$$\sqrt{8 - 4\sqrt{3}} = \sqrt{8 - 2\sqrt{12}}$$
$$= \sqrt{6 - 2\sqrt{12} + 2}$$
$$= \sqrt{(\sqrt{6})^2 - 2\sqrt{6}\sqrt{2} + (\sqrt{2})^2}$$

Recall that : $a^2 - 2ab + b^2 = (a - b)^2$

$$= \sqrt{(\sqrt{6} - \sqrt{2})^2}$$
$$= \sqrt{6} - \sqrt{2}$$

$\therefore \ a = 6$ and $b = 2$ as told $a > b$

[3 marks]

Answer 14

If you were going to differentiate you'd do this,

$$\frac{7\sqrt{p} - p^2}{\sqrt{p^3}} = \frac{7p^{\frac{1}{2}}}{p^{\frac{3}{2}}} - \frac{p^2}{p^{\frac{3}{2}}}$$
$$= 7p^{-1} - p^{\frac{1}{2}}$$
$$= \frac{7}{p} - \sqrt{p}$$

Which is by far the most elegant answer,
Other equivalent answers are likely, such as,

$$\frac{7 - p\sqrt{p}}{p}$$

[3 marks]

Answer 15

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$
$$2^x \times (2^2)^y = 2^{-\frac{3}{2}}$$
$$x + 2y = -\frac{3}{2}$$
$$2y = -\frac{3}{2} - x$$
$$y = -\frac{3}{4} - \frac{x}{2}$$

[3 marks]

Answer 16

$$2^7 \times 3 \times 5^4 = 2^6 \times 5^4 \times 2 \times 3$$

$$\sqrt{2^7 \times 3 \times 5^4} = \sqrt{2^6 \times 5^4} \times \sqrt{2 \times 3}$$

$$= 2^3 \times 5^2 \times \sqrt{6}$$

$$= 8 \times 25 \times \sqrt{6}$$

$$= 200\sqrt{6}$$

[2 marks]

Answer 17

$$\frac{1}{1 + \sqrt{2} + \sqrt{3}} = \frac{1}{(1 + \sqrt{2}) + \sqrt{3}}$$

$$= \frac{1}{(1 + \sqrt{2}) + \sqrt{3}} \times \frac{(1 + \sqrt{2}) - \sqrt{3}}{(1 + \sqrt{2}) - \sqrt{3}}$$

$$= \frac{(1 + \sqrt{2}) - \sqrt{3}}{(1 + \sqrt{2})^2 - 3}$$

$$= \frac{1 + \sqrt{2} - \sqrt{3}}{1 + 2\sqrt{2} + 2 - 3}$$

$$= \frac{1 + \sqrt{2} - \sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1 + \sqrt{2} - \sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{2} + 2 - \sqrt{6}}{4}$$

[4 marks]