4.2 Answers

A-Level Pure Mathematics : Year 1
Algebra of Surds and Indices II

Answer 1

$$\frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{7\sqrt{5}+5+7+\sqrt{5}}{5-\sqrt{5}+\sqrt{5}-1}$$

$$= \frac{12+8\sqrt{5}}{4}$$

$$= \frac{4\,(3+2\sqrt{5}\,)}{4}$$

$$= 3+2\sqrt{5}$$

[4 marks]

Answer 2

(a) 2 **(b)** $8x^9$

[1, 2 marks]

Answer 3

(i)

$$\left(\frac{18x^5y^2}{128x}\right)^{-\frac{1}{2}} = \left(\frac{9x^4y^2}{64}\right)^{-\frac{1}{2}}$$

$$= \sqrt{\left(\frac{64}{9x^4y^2}\right)}$$

$$= \frac{8}{3x^2y}$$

[3 marks]

(ii)

$$\left(\frac{8z^2}{12\sqrt{2}}\right)^3 = \left(\frac{2z^2}{3\sqrt{2}}\right)^3$$

$$= \frac{8z^6}{27 \times 2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2\sqrt{2}\,z^6}{27}$$

[3 marks]

(iii)

$$\left(\frac{49w^6}{27}\right)^{\frac{1}{2}} = \frac{7w^3}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{7\sqrt{3}\,w^3}{9}$$

[2 marks]

(iv)

$$\left(\frac{1+\sqrt{2}}{\sqrt{3}}\right)^{-2} = \left(\frac{\sqrt{3}}{1+\sqrt{2}}\right)^{2}$$

$$= \frac{3}{1+2\sqrt{2}+2}$$

$$= \frac{3}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{3(3-2\sqrt{2})}{9-8}$$

$$= 9-6\sqrt{2}$$

[4 marks]

Answer 4

$$\left(a+b\sqrt{c}\right)^2 = 6 + d\sqrt{c}$$

$$a^2 + 2ab\sqrt{c} + b^2 c = 6 + d\sqrt{c}$$

$$\therefore\ a^2 + b^2 c = 6 \quad \text{and} \quad 2ab = d$$

As a, b, c and d are positive integers

If $a = 2 \Rightarrow b = 1 \Rightarrow c = 2 \Rightarrow d = 4$

If $a = 1 \Rightarrow b = 1 \Rightarrow c = 5 \Rightarrow d = 2$

[3 marks]

Answer 5

It will help to think of $\sqrt{6}$ as $\sqrt{2} \times \sqrt{3}$ if doing this by mental arithmetic.

	$5\sqrt{2}$	$6\sqrt{3}$	$-2\sqrt{6}$
$4\sqrt{2}$	40	$24\sqrt{6}$	$-16\sqrt{3}$
$-2\sqrt{3}$	$-10\sqrt{6}$	-36	$12\sqrt{2}$
$3\sqrt{6}$	$30\sqrt{3}$	$54\sqrt{2}$	-36

$$\left(5\sqrt{2} + 6\sqrt{3} - 2\sqrt{6}\right)\left(4\sqrt{2} - 2\sqrt{3} + 3\sqrt{6}\right)$$

$$= -32 + 66\sqrt{2} + 14\sqrt{3} + 14\sqrt{6}$$

[5 marks]

Answer 6

$$64\sqrt{2} = 2^a$$

$$2^6 \times 2^{\frac{1}{2}} = 2^a$$

$$2^{6.5} = 2^a$$

$$a = 6.5$$

[2 marks]

Answer 7

$$12 - x\sqrt{24} = \frac{6x}{\sqrt{2}}$$

Multiply through by $\sqrt{2}$

$$12\sqrt{2} - x\sqrt{48} = 6x$$

$$6x + x\sqrt{16 \times 3} = 12\sqrt{2}$$

$$6x + 4x\sqrt{3} = 12\sqrt{2}$$

Divide through by 2

$$3x + 2x\sqrt{3} = 6\sqrt{2}$$

$$x\left(3 + 2\sqrt{3}\right) = 6\sqrt{2}$$

$$x = \frac{6\sqrt{2}}{3 + 2\sqrt{3}} \times \frac{3 - 2\sqrt{3}}{3 - 2\sqrt{3}}$$

$$= \frac{18\sqrt{2} - 12\sqrt{6}}{9 - 12}$$

$$= \frac{18\sqrt{2} - 12\sqrt{6}}{(-3)}$$

$$= 4\sqrt{6} - 6\sqrt{2}$$

or $2\sqrt{2}\left(2\sqrt{3} - 3\right)$

[5 marks]

Answer 8

(i) The $x \in R$ suggests that the initial intention was to allow all real numbers into the function, but then restrictions have been imposed. Firstly the taking of negative square roots has been avoided, which are not real. Secondly, the values of $x = 0$ and $x = 1$ have been excluded as these would cause a division by zero which is not defined.

[3 marks]

(ii)

$$\frac{x}{\sqrt{x} - x^2} = \frac{x}{\sqrt{x} - x^2} \times \frac{\sqrt{x} + x^2}{\sqrt{x} + x^2}$$ Key step !

$$= \frac{x\left(\sqrt{x} + x^2\right)}{x - x^4}$$

$$= \frac{x\left(\sqrt{x} + x^2\right)}{x\left(1 - x^3\right)}$$

$$= \frac{\sqrt{x} + x^2}{1 - x^3}$$

[3 marks]

Answer 9

$$\sqrt{18 + 2\sqrt{77}} = \sqrt{11 + 2\sqrt{11}\sqrt{7} + 7}$$

$$= \sqrt{\left(\sqrt{11}\right)^2 + 2\sqrt{11}\sqrt{7} + \left(\sqrt{7}\right)^2}$$

Recall that : $a^2 + 2ab + b^2 = (a + b)^2$

$$= \sqrt{\left(\sqrt{11} + \sqrt{7}\right)^2}$$

$$= \sqrt{11} + \sqrt{7}$$

$\therefore \; a = 11$ and $b = 7$ as told $a > b$

[3 marks]

Answer 10

Must make non-calculator method clear...

$$\sqrt{2^7 \times 3^3 \times 5^2} = \sqrt{2^6 \times 2 \times 3^2 \times 3 \times 5^2}$$

$$= 2^3 \times \sqrt{2} \times 3 \times \sqrt{3} \times 5$$

$$= 120\sqrt{6}$$

[2 marks]

Answer 11

(a)

$$\sqrt{50} - \sqrt{18} = \sqrt{25 \times 2} - \sqrt{9 \times 2}$$

$$= 5\sqrt{2} - 3\sqrt{2}$$

$$= 2\sqrt{2}$$

$\therefore \; a = 2$

[2 marks]

(b)

$$\frac{12\sqrt{3}}{\sqrt{50} - \sqrt{18}} = \frac{12\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{12\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{12\sqrt{6}}{4}$$

$$= 3\sqrt{6}$$

[3 marks]

Answer 12

$3.2 \times 10^{10k+1}$

[3 marks]

Answer 13

$$\left(4+\sqrt{p}\right)\left(3\sqrt{p}-1\right) = 17 + k\sqrt{p}$$
$$12\sqrt{p} - 4 + 3p - \sqrt{p} = 17 + k\sqrt{p}$$
$$3p - 4 + 11\sqrt{p} = 17 + k\sqrt{p}$$
$$\therefore\ k = 11 \text{ and } 3p - 4 = 17$$
$$3p = 21$$
$$p = 7$$

[3 marks]

Answer 14

$$\frac{9^x}{27^y} = \frac{1}{3\sqrt{3}}$$
$$\frac{\left(3^2\right)^x}{\left(3^3\right)^y} = 3^{-1.5}$$
$$3^{2x-3y} = 3^{-1.5}$$
$$\therefore 2x - 3y = -1.5$$
$$3y = 2x + \frac{3}{2}$$
$$y = \frac{2}{3}x + \frac{1}{2}$$

[4 marks]

Answer 15

(a)

$$2^x = \frac{1}{\sqrt{2}} \quad \text{and} \quad 2^y = 4\sqrt{2}$$
$$x = -\frac{1}{2} \quad \text{and} \quad y = \frac{5}{2}$$

[3 marks]

(b)

$$2^{y-x} = 2^3 = 8$$

[2 marks]

Answer 16

$$\frac{12\sqrt{3}}{\sqrt{2}+\sqrt{3}+\sqrt{5}} = \frac{12\sqrt{3}}{(\sqrt{2}+\sqrt{3})+\sqrt{5}} \times \frac{(\sqrt{2}+\sqrt{3})-\sqrt{5}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}}$$

$$= \frac{12\sqrt{3}\,(\sqrt{2}+\sqrt{3}-\sqrt{5})}{(\sqrt{2}+\sqrt{3})^2-5}$$

$$= \frac{12\sqrt{3}\,(\sqrt{2}+\sqrt{3}-\sqrt{5})}{2+2\sqrt{2}\sqrt{3}+3-5}$$

$$= \frac{12\sqrt{3}\,(\sqrt{2}+\sqrt{3}-\sqrt{5})}{2\sqrt{2}\sqrt{3}}$$

$$= \frac{6\,(\sqrt{2}+\sqrt{3}-\sqrt{5})}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{6\,(\sqrt{2}+\sqrt{3}-\sqrt{5})\sqrt{2}}{2}$$

$$= 3\sqrt{2}\,(\sqrt{2}+\sqrt{3}-\sqrt{5})$$

$$= 6+3\sqrt{6}-3\sqrt{10}$$

[5 marks]

Answer 17

$$s = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}}}$$

$$s^2 = 6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\ldots}}}}}$$

$$s^2 = 6+s$$

$$s^2-s-6 = 0$$

$$(s-3)(s+2) = 0$$

$$\therefore\ s = 3 \qquad \text{(Reject } s = -2 \text{ as not possible)}$$

[5 marks]