

Answers 11.2**A-Level Pure Mathematics : Year 2**
Differentiation IV**Answer 1**

$$(i) \quad y = \frac{2x^5}{15} \Rightarrow \frac{dy}{dx} = \frac{2 \times 5x^4}{15} \Rightarrow \frac{dy}{dx} = \frac{2x^4}{3}$$

[2 marks]

$$(ii) \quad y = \ln(2x^3 + 7) \Rightarrow \frac{dy}{dx} = \frac{6x^2}{2x^3 + 7}$$

[2 marks]

$$(iii) \quad y = \frac{5}{4x^2 - 3} \Rightarrow y = 5(4x^2 - 3)^{-1}$$
$$\frac{dy}{dx} = -5(4x^2 - 3)^{-2} \times 8x \Rightarrow \frac{dy}{dx} = -\frac{40x}{(4x^2 - 3)^2}$$

[2 marks]

$$(iv) \quad y = e^{\sqrt{x}} \Rightarrow y = e^{x^{\frac{1}{2}}}$$
$$\frac{dy}{dx} = e^{x^{\frac{1}{2}}} \times \frac{1}{2}x^{-\frac{1}{2}} \Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

[2 marks]**Answer 2**

$$y = 7x^2 \cos x$$
$$\frac{dy}{dx} = 7x^2 (-\sin x) + 14x \cos x$$
$$= 7x(2 \cos x - x \sin x)$$

[3 marks]**Answer 3**

$$y = \frac{\ln(4x)}{x^2}$$
$$\frac{dy}{dx} = \frac{x^2 \left(\frac{1}{4x} \times 4 \right) - 2x \ln(4x)}{(x^2)^2}$$
$$= \frac{x - 2x \ln(4x)}{x^4}$$
$$= \frac{x(1 - 2 \ln(4x))}{x^4}$$
$$= \frac{1 - 2 \ln(4x)}{x^3}$$

[3 marks]

Answer 4**(i)**

$$\begin{aligned}
 y &= \tan x \\
 &= \frac{\sin x}{\cos x} \\
 \frac{dy}{dx} &= \frac{\cos x \cos x - (-\sin x) \sin x}{\cos^2 x} \\
 &= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} \\
 &= \frac{1}{\cos^2 x} \\
 &= \sec^2 x \quad \square
 \end{aligned}$$

[4 marks]**(ii)**

$$\begin{aligned}
 y &= \tan^2 x \\
 \frac{dy}{dx} &= 2 \tan x \sec^2 x
 \end{aligned}$$

[2 marks]**Answer 5**

$$\text{(i) } \quad \text{(a) } \quad \frac{dx}{dt} = 12 - 3t^2 \quad \text{(b) } \quad \frac{dy}{dt} = 6t$$

$$\begin{aligned}
 \text{(c) } \quad \frac{dy}{dx} &= \frac{dy}{dt} \times \frac{dt}{dx} \\
 &= \frac{6t}{12 - 3t^2} \\
 &= \frac{2t}{4 - t^2}
 \end{aligned}$$

[2, 2, 2 marks]

$$\text{(ii) } \quad \text{When } t = 1, x = 11, y = 3 \quad \text{That is, } (11, 3)$$

[1 mark]

$$\text{(iii) } \quad \text{When } t = 1, \frac{dy}{dx} = \frac{2}{3}$$

[1 mark]

$$\text{(iv) } \quad y = mx + c \Rightarrow 3 = \frac{2}{3} \times 11 + c \Rightarrow c = \frac{9 - 22}{3} \Rightarrow c = -\frac{13}{3}$$

$$y = \frac{2}{3}x - \frac{13}{3}$$

[2 marks]

Answer 6

$$x^2 + 6xy - y^2 = 90$$

Differentiate with respect to x

$$2x + 6x \frac{dy}{dx} + 6y - 2y \frac{dy}{dx} = 0$$

Divide through by -2

$$-x - 3x \frac{dy}{dx} - 3y + y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} (y - 3x) = x + 3y$$

$$\frac{dy}{dx} = \frac{x + 3y}{y - 3x}$$

[6 marks]

Answer 7

(i)

t	-4	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	4
x	12	6	2	0	$-\frac{1}{4}$	0	$\frac{3}{4}$	2	6	12	20
y	20	12	6	2	$\frac{3}{4}$	0	$-\frac{1}{4}$	0	2	6	12

[3 marks]

(ii) See graph on the following page

[3 marks]

(iii) $\frac{dx}{dt} = 2t + 1 \quad \frac{dy}{dt} = 2t - 1 \quad \therefore \frac{dy}{dx} = \frac{2t - 1}{2t + 1}$

[4 marks]

(iv)

$$x = t^2 + t$$

Subtract $y = t^2 - t$

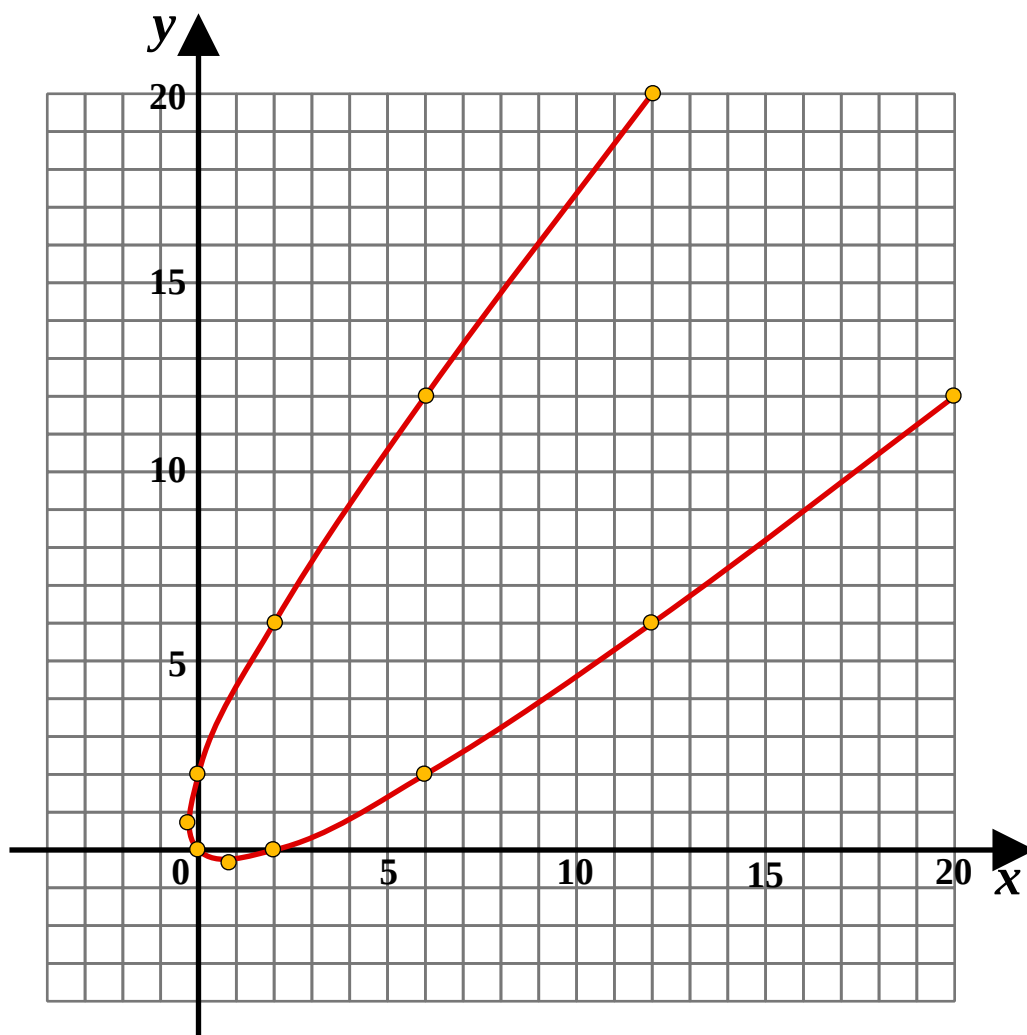
$$x - y = 2t$$

$$\frac{dy}{dx} = \frac{2t - 1}{2t + 1}$$

$$= \frac{x - y - 1}{x - y + 1}$$

[4 marks]

The Question 7, part (ii) graph



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