

Lesson 12

A-Level Pure Mathematics : Year 2

Differentiation IV

12.1 Test

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 50

Question 1

Differentiate each of the following;

(i) $y = \frac{4x^6}{15}$

[2 marks]

(ii) $y = \ln(x^4 - 1)$

[2 marks]

(iii) $y = \frac{7}{3x^2 - 2}$

[2 marks]

(iv) $y = e^{x^2}$

[2 marks]

Question 2

The product rule states that $(u v)' = u v' + u' v$

Use the rule to differentiate $y = 5x^2 \sin x$

[3 marks]

Question 3

The quotient rule states that $\left(\frac{u}{v}\right)' = \frac{v u' - v' u}{v^2}$

Use the rule to differentiate the following, simplifying your answer;

$$y = \frac{e^{4x}}{x^2}$$

[3 marks]

Question 4

- (i) Use derivatives of $\sin x$ and $\cos x$ to prove the derivative of $\cot x$ is $-\csc^2 x$

[4 marks]

- (ii) Hence, or otherwise, use the chain rule to differentiate;

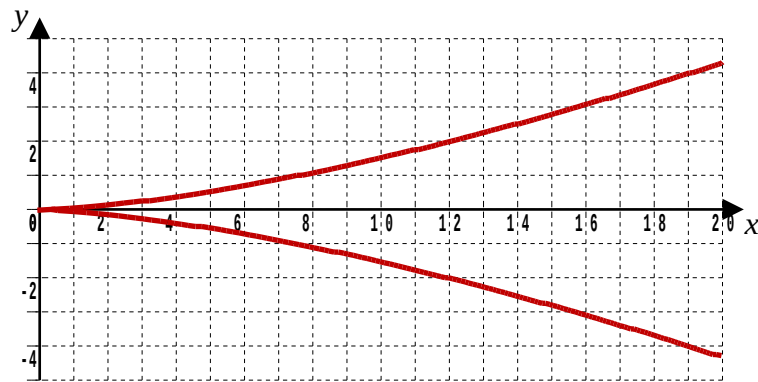
$$y = \cot^2 x$$

[2 marks]

Question 5

The semi-cubical parabola shown below, has parametric equations,

$$x = 12t^2, \quad y = 2t^3$$



(i) Find in terms of t ,

(a) $\frac{dx}{dt}$

(b) $\frac{dy}{dt}$

(c) $\frac{dy}{dx}$

[2, 2, 2 marks]

(ii) Write down the coordinates of the point on the curve that corresponds to the parameter t having the value 1

[1 mark]

(iii) What is the gradient of the curve at your part (ii) point ?

[1 mark]

(iv) By making use of your part (ii) and (iii) answers, determine the equation of the normal to the curve from the point at which $t = 1$

[3 marks]

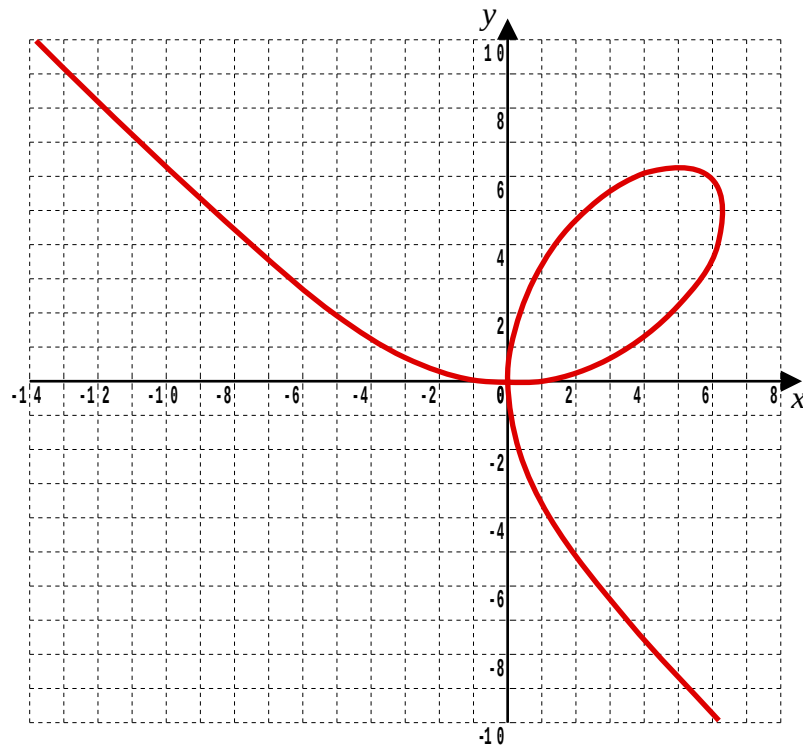
(v) Draw your part (iv) normal onto the graph above

[1 mark]

Question 6

The graph is of a famous curve known as the Folium of Descartes with equation,

$$y^3 + x^3 = 3xy$$



Differentiate Descartes' Folium by means of implicit differentiation.

Write your simplified answer in the form $\frac{dy}{dx} = f(x, y)$

[5 marks]

Question 7

The parametric equations of a curve are;

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}$$

- (i) Complete the following table by way of working out some points on the graph of this curve.

t	-5	-4	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{5}$
x									
y									

t	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	4	5
x									
y									

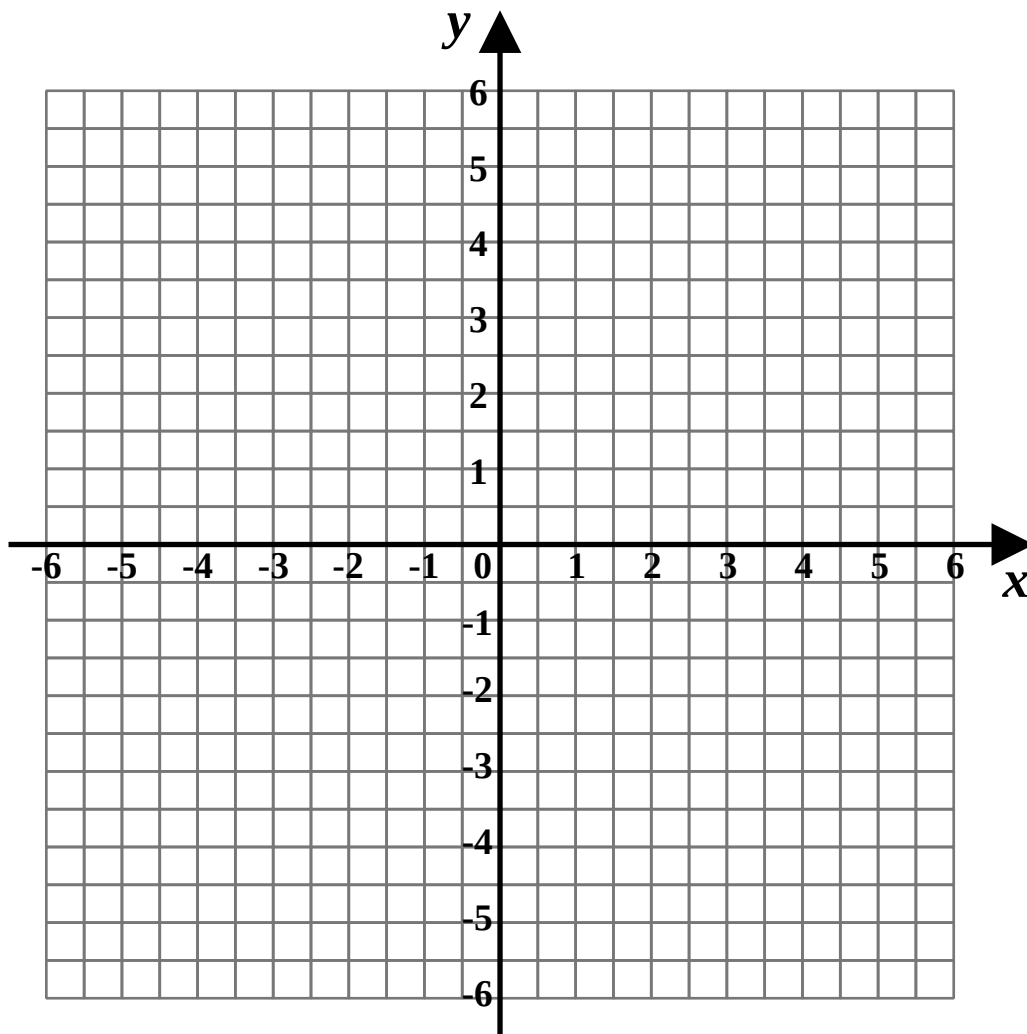
[4 marks]

- (ii) On the graph paper provided on the following page plot a graph of the curve.
(iii) What is the name of this type of curve ?

[1 mark]

- (iv) Find, in terms of t , an expression for the derivative of this curve.

[4 marks]



[4 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com