

12.2 Answers

A-Level Pure Mathematics : Year 2 Differentiation IV

Answer 1

$$(i) \quad y = \frac{4x^6}{15} \Rightarrow \frac{dy}{dx} = \frac{4 \times 6x^5}{15} \Rightarrow \frac{dy}{dx} = \frac{8x^5}{5}$$

[2 marks]

$$(ii) \quad y = \ln(x^4 - 1) \Rightarrow \frac{dy}{dx} = \frac{4x^3}{x^4 - 1}$$

[2 marks]

$$(iii) \quad y = \frac{7}{3x^2 - 2} \Rightarrow y = 7(3x^2 - 2)^{-1}$$
$$\frac{dy}{dx} = -7(3x^2 - 2)^{-2} \times 6x \Rightarrow \frac{dy}{dx} = -\frac{42x}{(3x^2 - 2)^2}$$

[2 marks]

$$(iv) \quad y = e^{x^2}$$
$$\frac{dy}{dx} = e^{x^2} \times 2x$$

[2 marks]

Answer 2

$$y = 5x^2 \sin x$$
$$\frac{dy}{dx} = 5x^2 (\cos x) + 10x \sin x$$
$$= 5x (x \cos x + 2 \sin x)$$

[3 marks]

Answer 3

$$y = \frac{e^{4x}}{x^2}$$
$$\frac{dy}{dx} = \frac{x^2 (e^{4x} \times 4) - 2x e^{4x}}{(x^2)^2}$$
$$= \frac{2x e^{4x} (2x - 1)}{x^4}$$
$$= \frac{2 e^{4x} (2x - 1)}{x^3}$$

[3 marks]

Answer 4**(i)**

$$\begin{aligned}
 y &= \cot x \\
 &= \frac{\cos x}{\sin x} \\
 \frac{dy}{dx} &= \frac{\sin x (-\sin x) - \cos x \cos x}{\sin^2 x} \\
 &= (-1) \frac{\sin^2 x + \cos^2 x}{\sin^2 x} \\
 &= -\frac{1}{\sin^2 x} \\
 &= -\csc^2 x \quad \square
 \end{aligned}$$

[4 marks]**(ii)**

$$\begin{aligned}
 y &= \cot^2 x \\
 \frac{dy}{dx} &= -2 \cot x \csc^2 x
 \end{aligned}$$

[2 marks]**Answer 5**

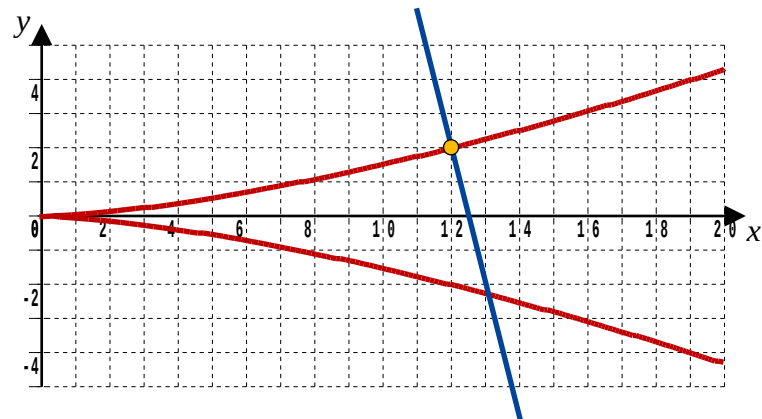
$$\begin{aligned}
 \text{(i)} \quad \text{(a)} \quad \frac{dx}{dt} &= 24t & \text{(b)} \quad \frac{dy}{dt} &= 6t^2 \\
 \text{(c)} \quad \frac{dy}{dx} &= \frac{dy}{dt} \times \frac{dt}{dx} \\
 &= \frac{6t^2}{24t} \\
 &= \frac{t}{4}
 \end{aligned}$$

[2, 2, 2 marks]**(ii)** When $t = 1$, $x = 12$, $y = 2$ That is, $(12, 2)$ **[1 mark]****(iii)** When $t = 1$, $\frac{dy}{dx} = \frac{1}{4}$ **[1 mark]**

(iv) $y = mx + c \Rightarrow 2 = -4 \times 12 + c \Rightarrow c = 50$
 $y = -4x + 50$

[2 marks]

(v)



[1 mark]

Answer 6

$$y^3 + x^3 = 3xy$$

Differentiate with respect to x

$$3y^2 \frac{dy}{dx} + 3x^2 = 3x \frac{dy}{dx} + 3y$$

Divide through by 3

$$y^2 \frac{dy}{dx} + x^2 = x \frac{dy}{dx} + y$$

$$\frac{dy}{dx} (y^2 - x) = y - x^2$$

$$\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$$

[5 marks]

Answer 7

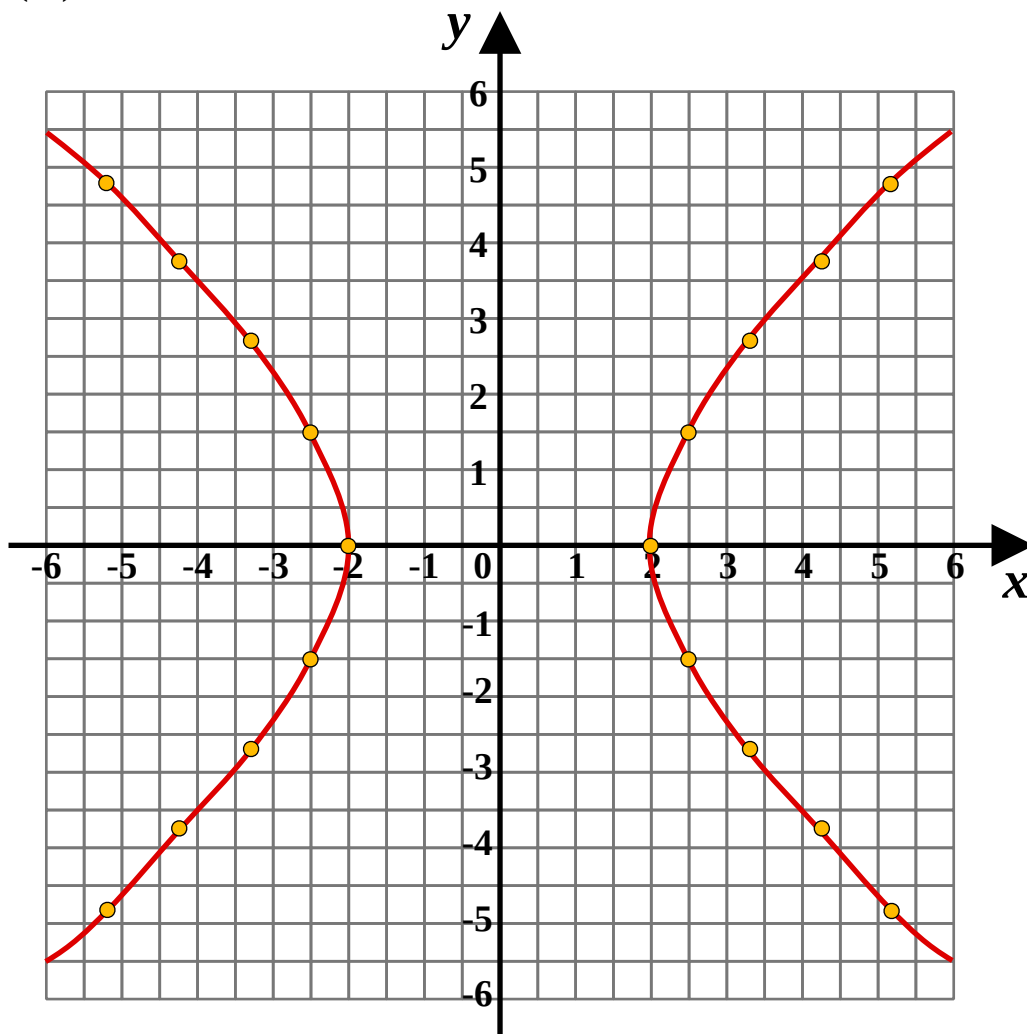
(i)

t	-5	-4	-3	-2	-1	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{5}$
x	-5.2	-4.25	-3.33	-2.5	-2	-2.5	-3.33	-4.25	-5.2
y	-4.8	-3.75	-2.66	-1.5	0	1.5	2.66	3.75	4.8

t	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	1	2	3	4	5
x	5.2	4.25	3.33	2.5	2	2.5	3.33	4.25	5.2
y	-4.8	-3.75	-2.66	-1.5	0	1.5	2.66	3.75	4.8

[4 marks]

(ii)



[3 marks]

(iii) A hyperbola, a hyperbolic curve

[1 mark]

(iv)

$$x = t + t^{-1} \Rightarrow \frac{dx}{dt} = 1 - t^{-2}$$

$$y = t - t^{-1} \Rightarrow \frac{dy}{dt} = 1 + t^{-2}$$

$$\frac{dy}{dx} = \frac{1 + t^{-2}}{1 - t^{-2}} \times \frac{t^2}{t^2}$$

$$= \frac{t^2 + 1}{t^2 - 1}$$

[4 marks]

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