13.2 Answers (Later Date Revision)

A-Level Pure Mathematics : Year 2
Differentiation IV

Answer 1

(i) $y = e^{5x}$ $\qquad \frac{dy}{dx} = 5e^{5x}$

(ii) $y = 4 \sin 6x$ $\qquad \frac{dy}{dx} = 24 \cos 6x$

(iii) $y = \cos^2 3x$ $\qquad \frac{dy}{dx} = -6 \cos 3x \sin 3x$

(iv) $y = \tan x$ $\qquad \frac{dy}{dx} = \sec^2 x$ $\qquad$ Use quotient rule if forgotten

(v) $y = \frac{1}{\cos x}$

$= \sec x$ $\qquad \frac{dy}{dx} = \sec x \tan x$ $\qquad$ Use chain rule if forgotten

(vi) $y = \ln\left(5x^3\right)$

$= \ln 5 + 3 \ln x$ $\qquad \frac{dy}{dx} = \frac{3}{x}$

(vii) $y = (\ln(5x))^3$ $\qquad \frac{dy}{dx} = 3(\ln(5x))^2 \times \frac{1}{x}$

[14 marks]

Answer 2

$$y = 3x e^{5x}$$

$$\frac{dy}{dx} = 3x \times 5e^{5x} + 3 \times e^{5x}$$

$$= 3e^{5x}(5x + 1)$$

Thus $A = 3$, $B = 5$ and $C = 1$

[3 marks]

Answer 3

$$y = \frac{\sin 3x}{2x^2}$$

$$\frac{dy}{dx} = \frac{2x^2 3\cos 3x - 4x \sin 3x}{4x^4}$$

$$= \frac{2x(3x \cos 3x - 2 \sin 3x)}{4x^4}$$

$$= \frac{3x \cos 3x - 2 \sin 3x}{2x^3}$$

[4 marks]

Answer 4

$$y = \left(1 + \cos^2 3x\right)^5$$

$$\frac{dy}{dx} = 5\left(1 + \cos^2 3x\right)^4 \times 2\cos 3x \times (-\sin 3x) \times 3$$

$$= -15\left(1 + \cos^2 3x\right)^4 \sin 6x$$

Thus $A = -15$, $B = 4$ and $C = 6$

[3 marks]

Answer 5

(i)

$$x = t - \cos^2 t \quad \Rightarrow \quad \frac{dx}{dt} = 1 - 2\cos t\ (-\sin t)$$

$$= 1 + \sin 2t$$

$$y = \sin^2 t \quad \Rightarrow \quad \frac{dy}{dt} = 2\sin t \cos t$$

$$= \sin 2t$$

$$\therefore \frac{dy}{dx} = \frac{\sin 2t}{1 + \sin 2t} \qquad \square$$

[4 marks]

(ii)

$$\text{When } t = \frac{\pi}{6}, \quad \frac{dy}{dx} = \frac{\sin\left(\frac{2\pi}{6}\right)}{1 - \sin\left(\frac{2\pi}{6}\right)}$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\right)}{1 - \left(\frac{\sqrt{3}}{2}\right)} \times \frac{\left(\frac{2}{1}\right)}{\left(\frac{2}{1}\right)}$$

$$= \frac{\sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}}$$

$$= \frac{2\sqrt{3} + 3}{4 - 3}$$

$$= 2\sqrt{3} + 3$$

Thus $m = 2$ and $n = 3$

[4 marks]

Answer 6

$y = \sin x \qquad \Rightarrow \qquad \frac{dy}{dx} = \cos x$

When $x = \frac{\pi}{3}$, $y = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ and $\frac{dy}{dx} = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

For the tangent, $y - mx + c$

$$\frac{\sqrt{3}}{2} = \frac{1}{2} \times \frac{\pi}{3} + c$$

$$c = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$

$$\therefore\ y = \frac{1}{2}x + \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$

[4 marks]

Answer 7

$$4x^3 + 5y^4 + 7xy = 0$$

$$12x^2 + 20y^3\frac{dy}{dx} + 7x\frac{dy}{dx} + 7y = 0$$

$$20y^3\frac{dy}{dx} + 7x\frac{dy}{dx} = -12x^2 - 7y$$

$$\frac{dy}{dx}\left(20y^3 + 7x\right) = -12x^2 - 7y$$

$$\frac{dy}{dx} = -\frac{12x^2 - 7y}{20y^3 + 7x}$$

[4 marks]