

2.4 Answers

A-Level Pure Mathematics : Year 2

Differentiation IV

2.4.1 Solutions to 2.1 Examples (Teaching Videos)

Example #1

$$x = 4 \sin \theta \Rightarrow \sin \theta = \frac{x}{4}$$

$$y = 6 \cos \theta \Rightarrow \cos \theta = \frac{y}{6}$$

$$\text{Trigonometric Identity : } \cos^2 \theta + \sin^2 \theta = 1$$

$$\therefore \left(\frac{y}{6}\right)^2 + \left(\frac{x}{4}\right)^2 = 1$$

$$\text{Multiply through by } 6^2 4^2$$

$$4^2 y^2 + 6^2 x^2 = 6^2 4^2$$

$$16 y^2 + 36 x^2 = 576$$

[4 marks]

Example #2

$$x = 4 t^2 \Rightarrow t^2 = \frac{x}{4}$$

$$\begin{aligned} y &= 16t(t^2 - 1) \Rightarrow y^2 = 256 t^2 (t^2 - 1)^2 \\ &= 256 \times \frac{x}{4} \left(\frac{x}{4} - 1 \right)^2 \\ &= 64x \left(\frac{x^2}{16} - \frac{x}{2} + 1 \right) \\ y^2 &= 4x^3 - 32x^2 + 64x \end{aligned}$$

[4 marks]

2.4.2 Solutions to 2.2 Exercise

Answer 1

$$x = 15 \sin \theta \Rightarrow \sin \theta = \frac{x}{15}$$

$$y = 20 \cos \theta \Rightarrow \cos \theta = \frac{y}{20}$$

$$\text{Trigonometric Identity : } \cos^2 \theta + \sin^2 \theta = 1$$

$$\therefore \left(\frac{y}{20}\right)^2 + \left(\frac{x}{15}\right)^2 = 1$$

$$\text{Multiply through by } 20^2 15^2$$

$$15^2 y^2 + 20^2 x^2 = 300^2$$

$$225 y^2 + 400 x^2 = 90000$$

$$9 y^2 + 16 x^2 = 3600$$

[4 marks]

Answer 2

$$\text{(i) } x = 8 t \quad \Rightarrow t = \frac{x}{8}$$

$$y = \frac{8}{t} \quad \Rightarrow 8 = yt \quad \Rightarrow 8 = y \times \frac{x}{8} \Rightarrow y = \frac{64}{x}$$

[3 marks]

$$\text{(ii) } x = \frac{20}{t} \quad \Rightarrow t^2 = \frac{400}{x^2}$$

$$y = t^2 \quad \Rightarrow y = \frac{400}{x^2}$$

[3 marks]

$$\text{(iii) } x = 14 t \quad \Rightarrow t = \frac{x}{14}$$

$$y = 7 t^2 \quad \Rightarrow y = \frac{7 x^2}{14^2} \Rightarrow y = \frac{x^2}{28}$$

[3 marks]

$$\text{(iv) } x = t - 2 \Rightarrow t^2 = (x + 2)^2$$

$$y = t^2 + 3 \quad \Rightarrow y = x^2 + 4x + 7$$

[3 marks]

Answer 3**(i)**

$$x = 2 \cos \theta \Rightarrow \cos \theta = \frac{x}{2}$$

$$y = 3 \sin \theta \Rightarrow \sin \theta = \frac{y}{3}$$

$$\text{Trigonometric Identity : } \cos^2 \theta + \sin^2 \theta = 1$$

$$\therefore \left(\frac{x}{2} \right)^2 + \left(\frac{y}{3} \right)^2 = 1$$

$$\text{Multiply through by } 2^2 3^2$$

$$3^2 x^2 + 2^2 y^2 = 6^2$$

$$9 x^2 + 4 y^2 = 36$$

[3 marks]**(ii)**

$$x = 3 \sec \theta \Rightarrow \sec \theta = \frac{x}{3}$$

$$y = 5 \tan \theta \Rightarrow \tan \theta = \frac{y}{5}$$

$$\text{Trigonometric Identity : } \cos^2 \theta + \sin^2 \theta = 1$$

$$\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\therefore 1 + \left(\frac{y}{5} \right)^2 = \left(\frac{x}{3} \right)^2$$

$$\text{Multiply through by } 5^2 3^2$$

$$5^2 3^2 + 3^2 y^2 = 5^2 x^2$$

$$225 + 9 y^2 = 25 x^2$$

$$25 x^2 - 9 y^2 = 225$$

[3 marks]

Answer 4

$$(i) \quad x = 2 + 3t \Rightarrow t = \frac{x-2}{3}$$

$$y = \frac{1}{t} \Rightarrow y = \frac{3}{x-2}$$

[3 marks]

$$(ii) \quad x = 3 + 2t \Rightarrow t = \frac{x-3}{2} \Rightarrow t^2 = \frac{x^2 - 6x + 9}{4}$$

$$y = 4t^2 - 9 \Rightarrow y = x^2 - 6x$$

[3 marks]**Answer 5**

$$x = 2 \cos \theta \Rightarrow x^2 = 4 \cos^2 \theta \Rightarrow \cos^2 \theta = \frac{x^2}{4}$$

$$y = 5 \sin 2\theta \Rightarrow y = 10 \sin \theta \cos \theta$$

$$y^2 = 100 \sin^2 \theta \cos^2 \theta$$

$$= 100 (1 - \cos^2 \theta) \cos^2 \theta$$

$$= 100 \left(1 - \frac{x^2}{4}\right) \left(\frac{x^2}{4}\right)$$

$$= 25x^2 \left(1 - \frac{x^2}{4}\right)$$

[6 marks]**Answer 6**

$$x = 2 \sin \theta \Rightarrow \sin^2 \theta = \frac{x^2}{4}$$

$$y = 1 - \cos 2\theta \Rightarrow y = 1 - (\cos^2 \theta - \sin^2 \theta)$$

$$= 1 - ((1 - \sin^2 \theta) - \sin^2 \theta)$$

$$= 1 - 1 + \sin^2 \theta + \sin^2 \theta$$

$$= 2 \sin^2 \theta$$

$$= \frac{x^2}{2}$$

[6 marks]

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