

2.4.3 Solutions to 2.3 Homework

A-Level Pure Mathematics : Year 2 Differentiation IV

Answer 1

$$(i) \quad x = \frac{12}{t} \Rightarrow t = \frac{12}{x}$$

$$y = t^2 + t \Rightarrow y = \left(\frac{12}{x}\right)^2 + \frac{12}{x} \Rightarrow y = \frac{144}{x^2} + \frac{12}{x}$$

[3 marks]

$$(ii) \quad x = 4\sqrt{t} \Rightarrow t^2 = \frac{x^4}{256}$$

$$y = 7t^2 \Rightarrow y = \frac{7x^4}{256}$$

[3 marks]

$$(iii) \quad x = e^{2t} \Rightarrow x^3 = e^{6t}$$

$$y = e^{6t} - 1 \Rightarrow y = x^3 - 1$$

[3 marks]

$$(iv) \quad x = \sqrt{t+1} \Rightarrow x^2 = t+1 \Rightarrow t = x^2 - 1$$

$$y = t^2 \Rightarrow y = (x^2 - 1)^2$$

[3 marks]

Answer 2

$$x = 35 \cos \theta^\circ \Rightarrow \cos \theta = \frac{x}{35}$$

$$y = 20 \sin \theta^\circ \Rightarrow \sin \theta = \frac{y}{20}$$

$$\text{Trigonometric Identity : } \cos^2 \theta + \sin^2 \theta = 1$$

$$\therefore \left(\frac{x}{35}\right)^2 + \left(\frac{y}{20}\right)^2 = 1$$

$$\text{Multiply through by } 35^2 20^2$$

$$20^2 x^2 + 35^2 y^2 = 700^2$$

$$400 x^2 + 1225 y^2 = 490000$$

$$\text{Divide through by 25}$$

$$16 x^2 + 49 y^2 = 19600$$

[4 marks]

Answer 3

$$x = \frac{1}{t-1} \Rightarrow t-1 = \frac{1}{x} \Rightarrow t = \frac{1}{x} + 1$$

$$y = \frac{1}{t+1} \Rightarrow t+1 = \frac{1}{y} \Rightarrow t = \frac{1}{y} - 1$$

$$\text{Thus, } \frac{1}{y} - 1 = \frac{1}{x} + 1$$

$$\frac{1}{y} = \frac{1}{x} + 2$$

$$\frac{1}{y} = \frac{1+2x}{x}$$

$$y = \frac{x}{1+2x}$$

That is, $a = 1$ and $b = 2$

[4 marks]