Answer 1

(a) $x = 4\sin^3 t \Rightarrow \dfrac{dx}{dt} = 12\sin^2 t\cos t$

$y = \cos 2t \Rightarrow \dfrac{dy}{dt} = -2\sin 2t \Rightarrow \dfrac{dy}{dt} = -4\sin t\cos t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{-4\sin t\cos t}{12\sin^2 t\cos t}$$

$$= -\frac{1}{3\sin t}$$

[4 marks]

(b) When $t = \dfrac{\pi}{6}$

$x = 4\sin^3\left(\dfrac{\pi}{6}\right) \Rightarrow x = 4 \times \left(\dfrac{1}{2}\right)^3 \Rightarrow x = \dfrac{1}{2}$

$y = \cos\left(\dfrac{\pi}{3}\right) \Rightarrow y = \dfrac{1}{2}$

$\dfrac{dy}{dx} = -\dfrac{1}{3\sin\left(\frac{\pi}{6}\right)} \Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{3} \times \dfrac{2}{1} \Rightarrow \dfrac{dy}{dx} = -\dfrac{2}{3}$

For the normal, the gradient will be $\dfrac{3}{2}$

$$y = mx + c$$

$$\frac{1}{2} = \frac{3}{2} \times \frac{1}{2} + c$$

$$c = -\frac{1}{4}$$

$\therefore$ Normal's equation is $y = \dfrac{3}{2}x - \dfrac{1}{4}$

[4 marks]

Answer 2

(a) $x = \tan^2 t \Rightarrow \dfrac{dx}{dt} = 2\tan t \sec^2 t$

$y = \sin t \Rightarrow \dfrac{dy}{dt} = \cos t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{\cos t}{2\tan t \sec^2 t}$$

$$= \frac{\cos t \cos t \cos^2 t}{2\sin t}$$

$$= \frac{\cos^4 t}{2\sin t} \qquad \therefore k = 4$$

[3 marks]

(b) As $\sin\left(\dfrac{\pi}{4}\right) = \cos\left(\dfrac{\pi}{4}\right)$ the gradient will be

$$\frac{1}{2} \times \left(\frac{\sqrt{2}}{2}\right)^3 = \frac{1}{2} \times \frac{2\sqrt{2}}{8}$$

$$= \frac{\sqrt{2}}{8}$$

[3 marks]

(c) **METHOD 1** $\cot^2 t + 1 = \csc^2 t$

$\sin t = y \qquad \tan^2 t = x$

$\dfrac{1}{\sin t} = \dfrac{1}{y} \qquad \dfrac{1}{\tan^2 t} = \dfrac{1}{x}$

$\csc t = \dfrac{1}{y} \qquad \cot^2 t = \dfrac{1}{x}$

$\csc^2 t = \dfrac{1}{y^2}$

$$\therefore \frac{1}{x} + 1 = \frac{1}{y^2}$$

$$\frac{1 + x}{x} = \frac{1}{y^2}$$

$$y^2 = \frac{x}{1 + x}$$

[4 marks]

(c) **METHOD 2** (Harry Marshall's solution)

$$y = \sin t \Rightarrow y^2 = \sin^2 t \qquad \text{so } \sin^2 t = y^2$$

$$y^2 = 1 - \cos^2 t \text{ so } \cos^2 t = 1 - y^2$$

$$\begin{aligned} x &= \tan^2 t \\ &= \frac{\sin^2 t}{\cos^2 t} \\ &= \frac{y^2}{1 - y^2} \\ x(1 - y^2) &= y^2 \\ x - xy^2 &= y^2 \\ y^2 + xy^2 &= x \\ y^2(1 + x) &= x \\ y^2 &= \frac{x}{1 + x} \end{aligned}$$

[4 marks]

Answer 3

(a) $x = 2\cos 2t \Rightarrow \dfrac{dx}{dt} = -4\sin 2t = -8\sin t\cos t$

$y = 6\sin t \Rightarrow \dfrac{dy}{dt} = 6\cos t$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{6\cos t}{-8\sin t\cos t}$$

$$= -\frac{3}{4\sin t}$$

When $t = \dfrac{\pi}{3}$ $\quad \dfrac{dy}{dx} = -\dfrac{3}{4\sin\left(\frac{\pi}{3}\right)}$

$$= -\frac{3 \times 2}{4\sqrt{3}}$$

$$= -\frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= -\frac{\sqrt{3}}{2}$$

[4 marks]

(b)

$$x = 2\cos^2 t - 2\sin^2 t$$

$$= 2\left(1 - \sin^2 t\right) - 2\sin^2 t$$

$$= 2 - 4\sin^2 t$$

$$= 2 - 4\left(\frac{y}{6}\right)^2$$

$$4\left(\frac{y}{6}\right)^2 = 2 - x$$

$$\frac{y^2}{9} = 2 - x$$

$$y^2 = 9(2 - x)$$

$$y = 3\sqrt{2 - x}$$

No $-\sqrt{\ }$ as the graph provided shows the other branch is not present for the given domain of t

[4 marks]