

7.4 Answers

A-Level Pure Mathematics : Year 2 Differentiation IV

7.4.1 Solution to 7.2 Example (Teaching Video)

$$y^3 = x^2 - y$$

$$3y^2 \frac{dy}{dx} = 2x - \frac{dy}{dx}$$

$$3y^2 \frac{dy}{dx} + \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} (3y^2 + 1) = 2x$$

$$\frac{dy}{dx} = \frac{2x}{3y^2 + 1}$$

7.4.1 Solutions to 7.3 Exercise

Answer 1

(i) $y^3 + 3y - x^3 - x = 130$

$$3y^2 \frac{dy}{dx} + 3 \frac{dy}{dx} - 3x^2 - 1 = 0$$

$$3y^2 \frac{dy}{dx} + 3 \frac{dy}{dx} = 3x^2 + 1$$

$$\frac{dy}{dx} (3y^2 + 3) = 3x^2 + 1$$

$$\frac{dy}{dx} = \frac{3x^2 + 1}{3y^2 + 3}$$

[5 marks]

(ii)
$$\begin{aligned} \text{LHS} &= y^3 + 3y - x^3 - x \\ &= 5^3 + 3 \times 5 - 2^3 - 2 \\ &= 125 + 15 - 8 - 2 \\ &= 130 \\ &= \text{RHS} \quad \therefore (2, 5) \text{ is on the curve } \square \end{aligned}$$

[2 marks]

(iii)
$$\begin{aligned} \frac{dy}{dx} &= \frac{3x^2 + 1}{3y^2 + 3} \\ &= \frac{3 \times 2^2 + 1}{3 \times 5^2 + 3} \\ &= \frac{13}{78} \\ &= \frac{1}{6} \quad \square \end{aligned}$$

[2 marks]

(iv)
$$\begin{aligned} y &= mx + c \\ 5 &= \frac{1}{6} \times 2 + c \\ c &= \frac{14}{3} \\ y &= \frac{1}{6}x + \frac{14}{3} \end{aligned}$$

$$x - 6y + 28 = 0$$

[3 marks]

Answer 2

$$\begin{aligned}y^4 &= \sin(x) + \sin(y) \\4y^3 \frac{dy}{dx} &= \cos(x) + \cos(y) \frac{dy}{dx} \\4y^3 \frac{dy}{dx} - \cos(y) \frac{dy}{dx} &= \cos(x) \\\frac{dy}{dx} (4y^3 - \cos(y)) &= \cos(x) \\\frac{dy}{dx} &= \frac{\cos(x)}{4y^3 - \cos(y)}\end{aligned}$$

[4 marks]

Answer 3

(a)

$$\begin{aligned}y^2 - 3y &= x^3 + 8 \\2y \frac{dy}{dx} - 3 \frac{dy}{dx} &= 3x^2 \\\frac{dy}{dx} (2y - 3) &= 3x^2 \\\frac{dy}{dx} &= \frac{3x^2}{2y - 3}\end{aligned}$$

[4 marks]

(b)

$$\begin{aligned}\text{When } y = 3, \quad 3^2 - 3 \times 3 &= x^3 + 8 \\9 - 9 &= x^3 + 8 \\x^3 &= -8 \\x &= -2\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{3x^2}{2y - 3} \\&= \frac{3 \times (-2)^2}{2 \times 3 - 3} \\&= \frac{12}{3} \\&= 4\end{aligned}$$

[3 marks]

Answer 4

$$3x^2 - 2y^2 + 2x - 3y + 5 = 0$$

$$6x - 4y \frac{dy}{dx} + 2 - 3 \frac{dy}{dx} = 0$$

$$4y \frac{dy}{dx} + 3 \frac{dy}{dx} = 6x + 2$$

$$\frac{dy}{dx} (4y + 3) = 6x + 2$$

$$\frac{dy}{dx} = \frac{6x + 2}{4y + 3}$$

$$\begin{aligned} \text{When } x = 0 \text{ and } y = 1, \frac{dy}{dx} &= \frac{6 \times 0 + 2}{4 \times 1 + 3} \\ &= \frac{2}{7} \end{aligned}$$

$$\text{For the normal, } m_N = -\frac{7}{2}$$

$$y = mx + c$$

$$1 = -\frac{7}{2} \times 0 + c$$

$$c = 1$$

$$y = -\frac{7}{2}x + 1$$

Multiply through by 2

$$7x + 2y - 2 = 0$$

[7 marks]