8.4 Answers

8.4.1 Solution to 8.2 Example (Teaching Video)

$$4x^2y^3 = 4 + x^2 - y^2$$

$$(4x^2)(y^3) = 4 + x^2 - y^2$$

$$(4x^2) \times 3y^2\frac{dy}{dx} + 8x \times (y^3) = 2x - 2y\frac{dy}{dx}$$

$$12x^2y^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 2x - 8xy^3$$

Divide through by 2

$$6x^2y^2\frac{dy}{dx} + y\frac{dy}{dx} = x - 4xy^3$$

$$\frac{dy}{dx}\left(6x^2y^2 + y\right) = x - 4xy^3$$

$$\frac{dy}{dx} = \frac{x - 4xy^3}{6x^2y^2 + y}$$

$$= \frac{x(1 - 4y^3)}{y(6x^2y + 1)}$$

[4 marks]

8.4.2 Solutions to 8.3 Exercise

Answer 1

(i)

$$3x^2 y = 4x - y^3$$

$$3x^2 \frac{dy}{dx} + 6xy = 4 - 3y^2 \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(3x^2 + 3y^2\right) = 4 - 6xy$$

$$\frac{dy}{dx} = \frac{4 - 6xy}{3x^2 + 3y^2}$$

[6 marks]

(ii)

$$\text{LHS} = 3x^2 y = 3 \times 1^2 \times 1 = 3$$

$$\text{RHS} = 4x - y^3 = 4 \times 1 - 1^3 = 3 = \text{LHS}$$

$\therefore\ Q(1,\ 1)$ is on the curve □

[2 marks]

(iii)

$$\frac{dy}{dx} = \frac{4 - 6xy}{3x^2 + 3y^2} = \frac{4 - 6 \times 1 \times 1}{3 \times 1^2 + 3 \times 1^2} = \frac{-2}{6} = -\frac{1}{3} \quad \square$$

[2 marks]

(iv)

$$y = mx + c$$

$$1 = -\frac{1}{3} \times 1 + c$$

$$c = \frac{4}{3}$$

$$y = -\frac{1}{3}x + \frac{4}{3}$$

$$x + 3y - 4 = 0$$

[2 marks]

Answer 2

(i)

$$2xy = y^3 + 2x - 3x^2$$

$$2x\frac{dy}{dx} + 2y = 3y^2\frac{dy}{dx} + 2 - 6x$$

$$\frac{dy}{dx}\left(2x - 3y^2\right) = 2 - 6x - 2y$$

$$\frac{dy}{dx} = \frac{2 - 6x - 2y}{2x - 3y^2}$$

[6 marks]

(ii)

$$\begin{aligned} \text{LHS} &= 2xy \\ &= 2 \times (-2) \times 2 \\ &= -8 \end{aligned} \qquad \begin{aligned} \text{RHS} &= y^3 + 2x - 3x^2 \\ &= 2^3 + 2 \times (-2) - 3 \times (-2)^2 \\ &= 8 - 4 - 12 \\ &= -8 \\ &= \text{LHS} \end{aligned}$$

$$\therefore\ R(-2,\ 2) \text{ is on the curve } \square$$

[2 marks]

(iii) The only possibility is (1, – 1)

$$\begin{aligned} \text{LHS} &= 2xy \\ &= 2 \times 1 \times (-1) \\ &= -2 \end{aligned} \qquad \begin{aligned} \text{RHS} &= y^3 + 2x - 3x^2 \\ &= (-1)^3 + 2 \times 1 - 3 \times 1^2 \\ &= -1 + 2 - 3 \\ &= -2 \\ &= \text{LHS} \end{aligned}$$

$$\therefore\ \left(1, -1\right) \text{ is on the curve}$$

[2 marks]

(iv) At (1, – 1) $\dfrac{dy}{dx} = \dfrac{2 - 6x - 2y}{2x - 3y^2}$

$$= \frac{2 - 6 \times 1 - 2 \times (-1)}{2 \times 1 - 3 \times (-1)^2}$$

$$= 2$$

[2 marks]

(v)

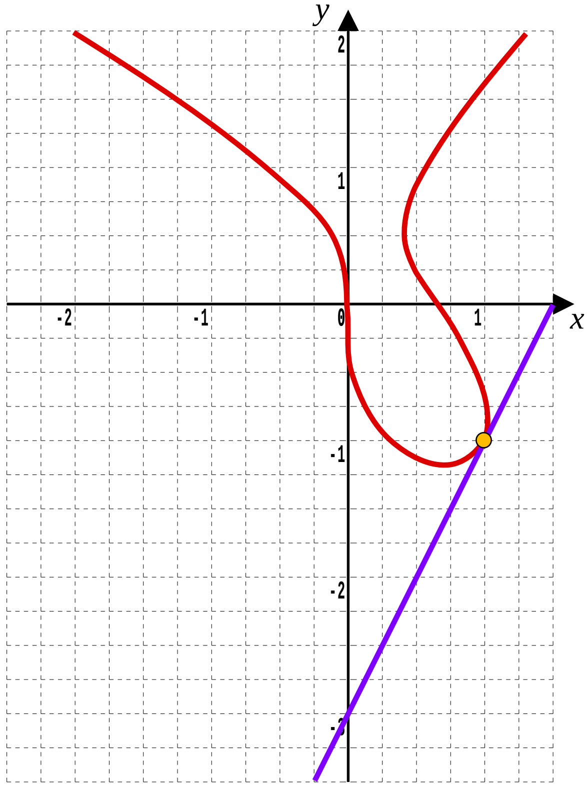
$$y = mx + c$$

$$-1 = 2 \times 1 + c \Rightarrow c = -3$$

$$y = 2x - 3$$

[2 marks]

(vi)



[3 marks]

Answer 3

(a)

$$2x + 3y^2 + 3x^2 y = 4x^2$$

Differentiate with respect to x

$$2 + 6y\frac{dy}{dx} + 3x^2\frac{dy}{dx} + 6xy = 8x$$

$$\frac{dy}{dx}\left(6y + 3x^2\right) = 8x - 6xy - 2$$

$$\frac{dy}{dx} = \frac{8x - 6xy - 2}{6y + 3x^2}$$

At $P(-1,\ 1)$ this becomes, $\frac{dy}{dx} = \frac{-8 + 6 - 2}{6 + 3}$

$$= -\frac{4}{9}$$

[5 marks]

(b)

Thus, $m_N = \frac{9}{4} \Rightarrow y = mx + c$

$$1 = \frac{9}{4} \times (-1) + c$$

$$c = \frac{13}{4}$$

$$y = \frac{9}{4}x + \frac{13}{4}$$

$$9x - 4y + 13 = 0$$

[3 marks]

Answer 4

$$3x^2 + 4y^2 - 2x + 6xy - 5 = 0$$

Differentiate with respect to x

$$6x + 8y\frac{dy}{dx} - 2 + 6x\frac{dy}{dx} + 6y = 0$$

$$\frac{dy}{dx}(8y + 6x) = 2 - 6x - 6y$$

Divide through by 2

$$\frac{dy}{dx}(4y + 3x) = 1 - 3x - 3y$$

$$\frac{dy}{dx} = \frac{1 - 3x - 3y}{4y + 3x}$$

At $(1, -2)$ this becomes, $\frac{dy}{dx} = \frac{1 - 3 + 6}{-8 + 3}$

$$= -\frac{4}{5}$$

$$y = mx + c$$

$$-2 = -\frac{4}{5} \times 1 + c$$

$$c = -\frac{6}{5}$$

$$\therefore\ y = -\frac{4}{5}x - \frac{6}{5}$$

$$4x + 5y + 6 = 0$$

[7 marks]

Answer 5

$$x^2 + 2xy - 3y^2 + 16 = 0$$

Differentiate with respect to x

$$2x + 2x\frac{dy}{dx} + 2y - 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(6y - 2x) = 2x + 2y$$

Divide through by 2

$$\frac{dy}{dx}(3y - x) = x + y$$

$$\frac{dy}{dx} = \frac{x + y}{3y - x}$$

For $\frac{dy}{dx} = 0, \quad x + y = 0 \Rightarrow y = -x$

Replace y in the original equation with $-x$ and solve for x

$$x^2 + 2x(-x) - 3(-x)^2 + 16 = 0$$

$$x^2 - 2x^2 - 3x^2 + 16 = 0$$

$$4x^2 = 16$$

$$x = \pm 2$$

So the points where $\frac{dy}{dx} = 0$ are $(-2, 2), (2, -2)$

[7 marks]

Answer 6

(a)

$$x^3 - 4y^2 = 12xy$$

When $x = -8$, $(-8)^3 - 4y^2 = 12(-8)y$

$$-512 - 4y^2 = -96y$$

Divide through by -4

$$128 + y^2 = 24y$$

$$y^2 - 24y + 128 = 0$$

$$(y - 16)(y - 8) = 0$$ or use the Q-Formula

Either $y = 16$ or $y = 8$

$\therefore$ Coordinates are, $(-8, 8)$, $(-8, 16)$

[3 marks]

(b)

$$x^3 - 4y^2 = 12xy$$

Differentiate with respect to x

$$3x^2 - 8y\frac{dy}{dx} = 12x\frac{dy}{dx} + 12y$$

$$\frac{dy}{dx}\left(12x + 8y\right) = 3x^2 - 12y$$

$$\frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y}$$

For $(-8, 8)$, $\dfrac{dy}{dx} = \dfrac{3 \times (-8)^2 - 12 \times 8}{12 \times (-8) + 8 \times 8}$

$$= \frac{192 - 96}{-96 + 64}$$

$$= -3$$

For $(-8, 16)$, $\dfrac{dy}{dx} = \dfrac{3 \times (-8)^2 - 12 \times 16}{12 \times (-8) + 8 \times 16}$

$$= \frac{192 - 192}{-96 + 128}$$

$$= 0$$

[6 marks]

Answer 7
(a)

$$3x^2 - y^2 + xy = 4$$

Differentiate with respect to x

$$6x - 2y\frac{dy}{dx} + x\frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx}\left(2y - x \right) = 6x + y$$

$$\frac{dy}{dx} = \frac{6x + y}{2y - x}$$

At P and Q, $\quad \frac{dy}{dx} = \frac{8}{3}$

$$\frac{6x + y}{2y - x} = \frac{8}{3}$$

$$3(6x + y) = 8(2y - x)$$

$$18x + 3y = 16y - 8x$$

$$26x = 13y$$

$$y = 2x$$

$$y - 2x = 0 \qquad \square$$

[6 marks]

(b)

At P and at Q, $y = 2x$

$$3x^2 - (2x)^2 + x(2x) = 4$$

$$3x^2 - 4x^2 + 2x^2 = 4$$

$$x^2 = 4$$

$$x = \pm 2$$

Thus, $\left(-2, -4 \right), \left(2, 4 \right)$

[3 marks]