

9.5 Answers

A-Level Pure Mathematics : Year 2

Differentiation IV

9.5.1 Solution to 9.3 A Beautiful Curve (Teaching Video)

$$3^y 3^x = x^6 + y^6$$

$$(3^y)(3^x) = x^6 + y^6$$

Differentiate with respect to x

$$(3^y) 3^x \ln 3 + 3^y \ln 3 \frac{dy}{dx} (3^x) = 6x^5 + 6y^5 \frac{dy}{dx}$$

$$\frac{dy}{dx} (3^y 3^x \ln 3 - 6y^5) = 6x^5 - 3^y 3^x \ln 3$$

$$\frac{dy}{dx} = \frac{6x^5 - 3^y 3^x \ln 3}{3^y 3^x \ln 3 - 6y^5}$$

[7 marks]

9.5.2 Solutions to 9.4 Exercise

Answer 1

(i) $3^x + 2xy = y$

Differentiate with respect to x

$$3^x \ln 3 + 2x \frac{dy}{dx} + 2y = \frac{dy}{dx}$$

$$\frac{dy}{dx} (1 - 2x) = 3^x \ln 3 + 2y$$

$$\frac{dy}{dx} = \frac{3^x \ln 3 + 2y}{1 - 2x}$$

[7 marks]

(ii) LHS = $3^x + 2xy$ RHS = y

$$= 3^2 + 2 \times 2 \times (-3) \qquad = -3$$

$$= 9 - 12 \qquad = \text{LHS}$$

$$= -3 \qquad \therefore (2, -3) \text{ is on the curve}$$

[2 marks]

(iii)
$$\begin{aligned} \frac{dy}{dx} &= \frac{3^x \ln 3 + 2y}{1 - 2x} \\ &= \frac{3^2 \ln 3 + 2 \times (-3)}{1 - 2 \times 2} \\ &= 2 - 3 \ln 3 \end{aligned}$$

[2 marks]

Answer 2**(i)**

$$y e^{3x} + 3x = y^3$$

Differentiate with respect to x

$$y 3 e^{3x} + \frac{dy}{dx} e^{3x} + 3 = 3 y^2 \frac{dy}{dx}$$

$$\frac{dy}{dx} (3 y^2 - e^{3x}) = 3 y e^{3x} + 3$$

$$\frac{dy}{dx} = \frac{3 y e^{3x} + 3}{3 y^2 - e^{3x}}$$

[7 marks]**(ii)**

$$\text{At } (0, 0), \quad \frac{dy}{dx} = \frac{3 y e^{3x} + 3}{3 y^2 - e^{3x}}$$

$$= -3$$

$$\therefore m_N = \frac{1}{3}$$

$$\therefore \text{Equation of normal is } y = \frac{1}{3} x$$

See graph on the following page

[3 marks]**(iii)**

$$\text{At } (0, 1), \quad \frac{dy}{dx} = \frac{3 y e^{3x} + 3}{3 y^2 - e^{3x}}$$

$$= 3$$

$$\therefore m_N = -\frac{1}{3}$$

$$\therefore \text{Equation of normal is } y = -\frac{1}{3} x + 1$$

See graph on the following page

[3 marks]**(iv)**

$$\frac{1}{3} x = -\frac{1}{3} x + 1$$

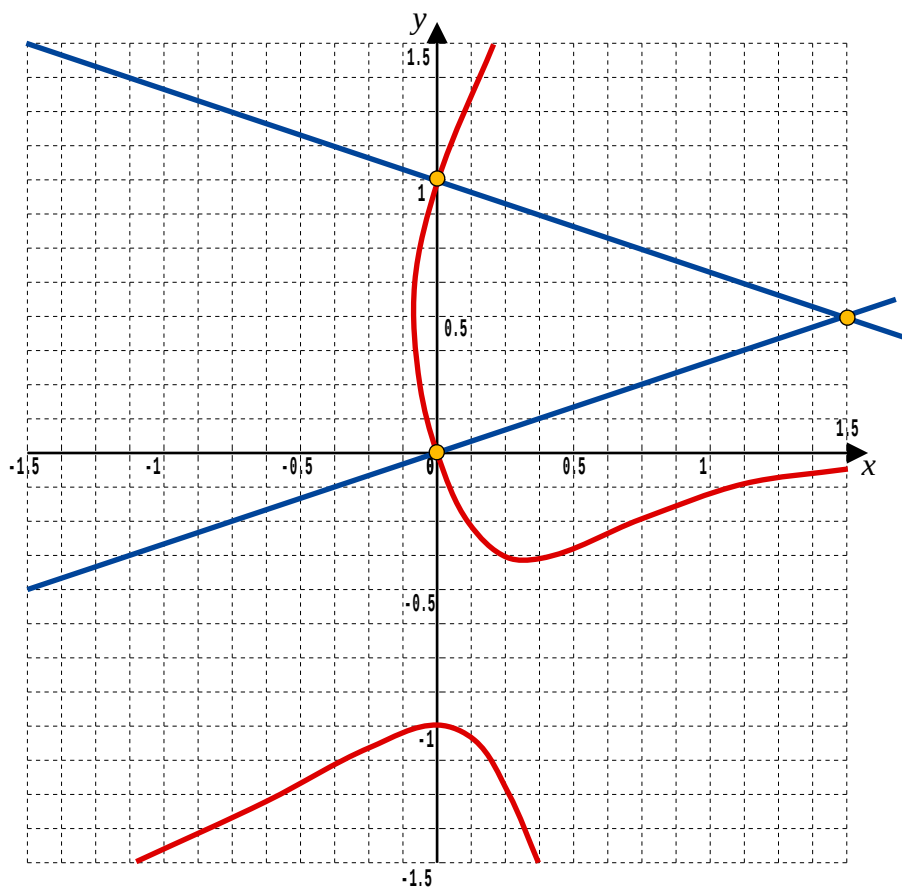
$$\frac{2}{3} x = 1$$

$$x = \frac{3}{2} \text{ and } y = \frac{1}{2}$$

See graph on the following page

[3 marks]

Graph for Answer 2



Answer 3

$$3^x + xy = x + y^2$$

Differentiate with respect to x

$$3^x \ln 3 + x \frac{dy}{dx} + y = 1 + 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} (x - 2y) = 1 - 3^x \ln 3 - y$$

$$\frac{dy}{dx} = \frac{1 - 3^x \ln 3 - y}{x - 2y}$$

$$\begin{aligned} \text{At the point } P(4, 11), \quad \frac{dy}{dx} &= \frac{1 - 3^4 \ln 3 - 11}{4 - 22} \\ &= \frac{-10 - 81 \ln 3}{-18} \\ &= \frac{5}{9} + \frac{9}{2} \ln 3 \end{aligned}$$

[6 marks]

Answer 4

$$\begin{aligned}2^x + y^2 &= 2xy \\2^x \ln 2 + 2y \frac{dy}{dx} &= 2x \frac{dy}{dx} + 2y \\ \frac{dy}{dx} (2y - 2x) &= 2y - 2^x \ln 2 \\ \frac{dy}{dx} &= \frac{2y - 2^x \ln 2}{2y - 2x} \\ \text{At } (3, 2), \frac{dy}{dx} &= \frac{2 \times 2 - 2^3 \ln 2}{2 \times 2 - 2 \times 3} \\ &= \frac{4 - 8 \ln 2}{4 - 6} \\ &= 4 \ln 2 - 2\end{aligned}$$

[7 marks]