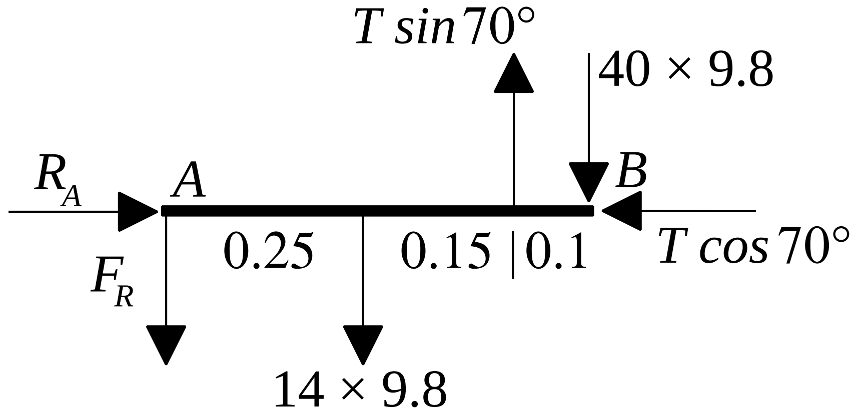
2.3 Answers

2.3.1 Solution to the Introductory 2.1 Example



(i) Taking moments about A,

$$T\,sin\,70 \times 0.4 = 14 \times 9.8 \times 0.25 + 40 \times 9.8 \times 0.5$$
$$0.376\,T = 34.3 + 196$$
$$T = 613\,\text{N}$$

(ii) Balancing forces vertically,

$$F_R + 14 \times 9.8 + 40 \times 9.8 = T\,sin\,70$$
$$F_R + 137 + 392 = 613\,sin\,70$$
$$F_R = 47.0\,\text{N}$$

(iii) Balancing forces horizontally,

$$R_A = T\,cos\,70$$
$$= 613 \times cos 70$$
$$= 210\,\text{N}$$

(iv) Being told “her boots are about to slip” implied limiting equilibrium,

$$F_R = \mu\,R_A$$
$$\mu = \frac{47}{210}$$
$$= 0.224$$

(v) Overall force exerted by the rock face on her boots is given by Pythagoras',

$$\sqrt{(F_R)^2 + (R_A)^2} = \sqrt{47^2 + 210^2}$$
$$= 215\,\text{N}$$

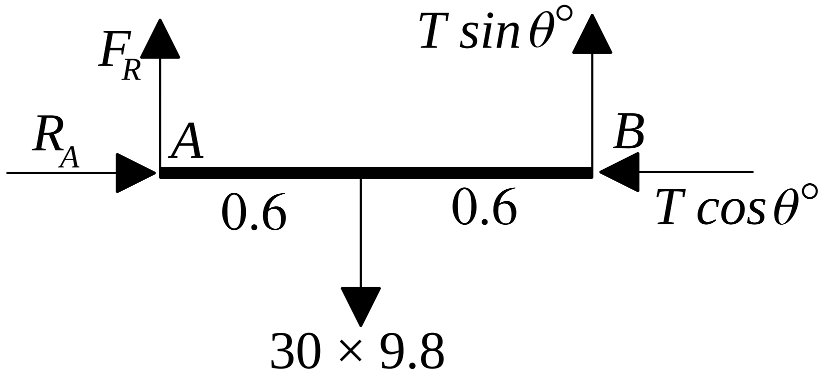
and simple trigonometry,

$$\theta = arctan\left(\frac{F_R}{R_A}\right)$$
$$= arctan\left(\frac{47}{210}\right)$$
$$= 12.6° \quad \text{below the horizontal}$$

2.3.2 Solutions to 2.2 Exercise

Answer 1

(i)



(ii)

$$\tan\theta = \frac{5}{12} \qquad \cos\theta = \frac{12}{13} \qquad \sin\theta = \frac{5}{13}$$

(iii) Taking moments about A,

$$T \sin\theta \times 1.2 = 30 \times g \times 0.6$$

$$T \times \frac{5}{13} \times 1.2 = 18g$$

$$T = \frac{18 \times 13}{5 \times 1.2} g$$

$$= 39\,g \text{ Newtons} \qquad \square$$

(iv) Balancing forces horizontally,

$$R_A = T\cos\theta$$

$$= 39\,g \times \frac{12}{13}$$

$$= 36\,g \text{ N}$$

Balancing forces vertically,

$$F_R + T\sin\theta = 30\,g$$

$$F_R = 30\,g - 39\,g \times \frac{5}{13}$$

$$= 15\,g \text{ N}$$

Considering friction,

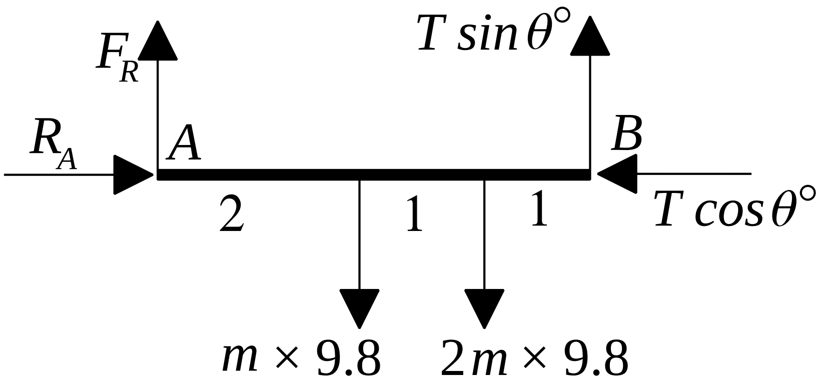
$$F_R = \mu R_A$$

$$\mu = \frac{15\,g}{36\,g}$$

$$= \frac{5}{12} \qquad (0.42)$$

Answer 2

(a) First a working diagram,



$$tan\ \theta = \frac{3}{4} \qquad cos\ \theta = \frac{4}{5} \qquad sin\ \theta = \frac{3}{5}$$

Taking moments about A,

$$T\ sin\ \theta \times 4 = m \times g \times 2 + 2m \times g \times 3$$

$$T \times \frac{3}{5} \times 4 = 2mg + 6mg$$

$$T = \frac{10}{3}\,mg \qquad (3.33mg)$$

(b) Balancing forces horizontally,

$$R_A = T\ cos\ \theta$$

$$= \frac{10}{3}\,mg \times \frac{4}{5}$$

$$= \frac{8}{3}\,mg \qquad \square$$

(c) Balancing forces vertically,

$$F_R + T\ sin\ \theta = m \times g + 2m \times g$$

$$F_R = 3\,mg - \frac{10}{3}\,mg \times \frac{3}{5}$$

$$= mg$$

Considering friction,

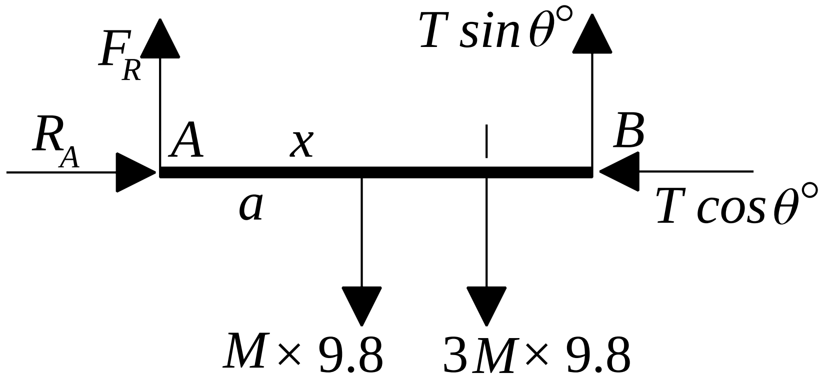
$$F_R = \mu\,R_A$$

$$\mu = \frac{mg}{\frac{8}{3}\,mg}$$

$$= \frac{3}{8} \qquad (0.38)$$

Answer 3

(a) First a working diagram,



$$\tan\theta = \frac{3}{4} \qquad \cos\theta = \frac{4}{5} \qquad \sin\theta = \frac{3}{5}$$

Taking moments about A,

$$T \sin\theta \times 2a = M \times g \times a + 3M \times g \times x$$

$$T \times \frac{3}{5} \times 2a = Mga + 3Mgx$$

$$6Ta = 5Mg\ (a + 3x)$$

$$T = \frac{5Mg\ (3x + a)}{6a} \qquad \square$$

(b) Balancing forces horizontally,

$$R_A = T \cos\theta$$

$$2Mg = \frac{5Mg\ (3x + a)}{6a} \times \frac{4}{5}$$

$$30a \times 2Mg = 20Mg (3x + a)$$

$$60Mga = 60Mgx + 20Mga$$

$$40Mga = 60Mgx$$

$$60x = 40a$$

$$x = \frac{2}{3}a$$

(c) Balancing forces vertically,

$$F_R + T \sin\theta = Mg + 3Mg$$

$$F_R = 4Mg - \frac{5Mg\,(3x + a)}{6a} \times \frac{3}{5}$$

$$= 4Mg - \frac{Mg\,(3 \times \tfrac{2}{3}a + a)}{2a}$$

$$= 4Mg - \frac{Mg \times 3a}{2a}$$

$$= \frac{8Mg}{2} - \frac{3Mg}{2}$$

$$= \frac{5Mg}{2}$$

$$\tan\beta = \frac{F_R}{R_A}$$

$$= \frac{5Mg}{2 \times 2Mg}$$

$$= \frac{5}{4}$$

(d)

$$T \leqslant 5Mg$$

$$\frac{5Mg\,(3x + a)}{6a} \leqslant 5Mg$$

$$\frac{(3x + a)}{6a} \leqslant 1$$

$$3x + a \leqslant 6a$$

$$3x \leqslant 5a$$

$$x \leqslant \frac{5}{3}a$$

$\therefore$ For the rope not to break, the block must be less than $\frac{5a}{3}$ from A