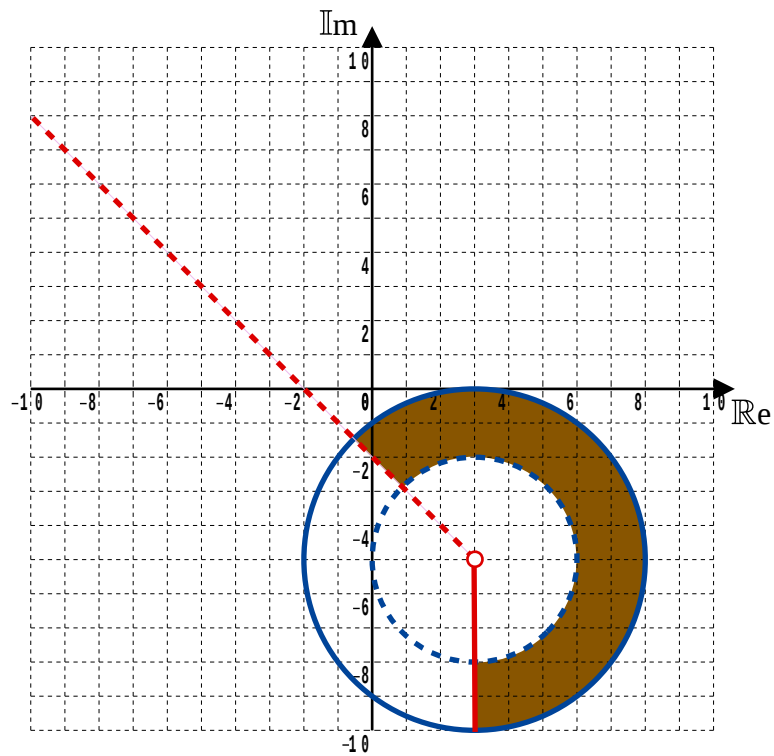


10.3 Answers

Further A-Level Pure Mathematics : Core 1 Complex Numbers I

10.3.1 Solution to 10.1 Teaching Video Example

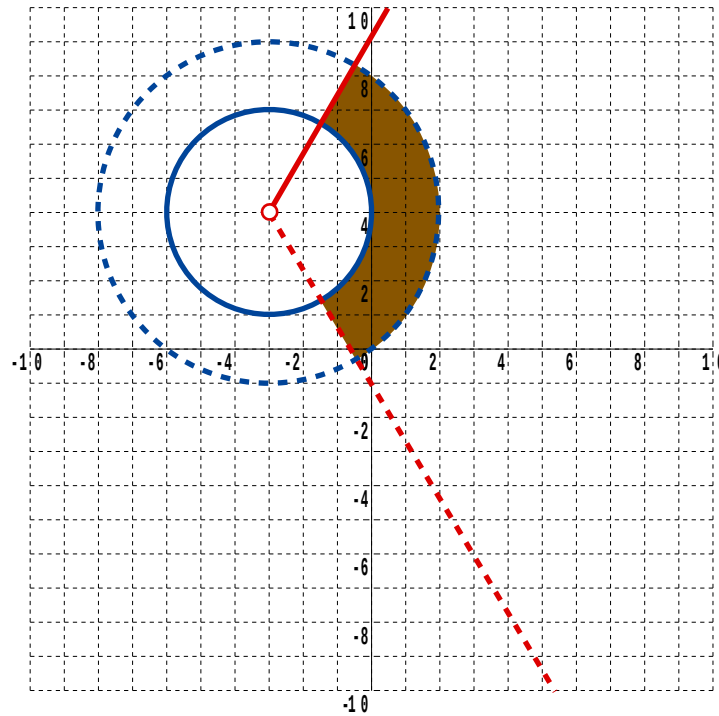


[4 marks]

10.3.2 Solutions to 10.2 Exercise

Answer 1

(i)



[4 marks]

- (ii) The perimeter is made up of one 120° arc of a circle of radius 3, another 120° arc of a circle of radius 5 and two straight radial lines between the differences in the radii of the concentric circles.

$$\begin{aligned} \text{Perimeter} &= 3 \times \frac{2\pi}{3} + 5 \times \frac{2\pi}{3} + 2 + 2 \\ &= \frac{16\pi}{3} + 4 \quad (20.8 \text{ units}) \end{aligned}$$

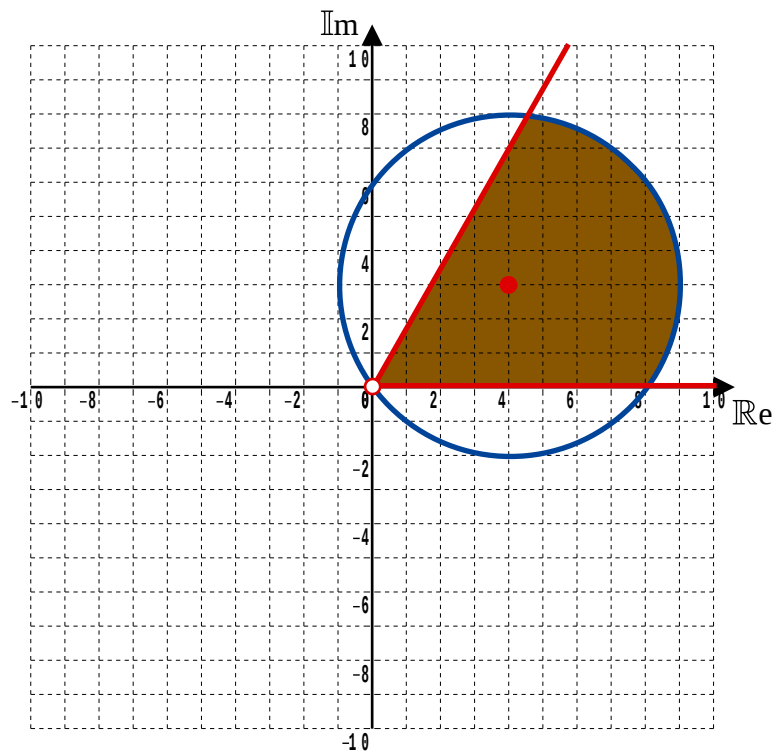
[2 marks]

- (iii) The area is made up of one 120° segment of a circle of radius 3, subtracted from another 120° segment of a circle of radius 5.

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 5^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 3^2 \times \frac{2\pi}{3} \\ &= \frac{25\pi - 9\pi}{3} \\ &= \frac{16\pi}{3} \quad (16.8 \text{ units}^2) \end{aligned}$$

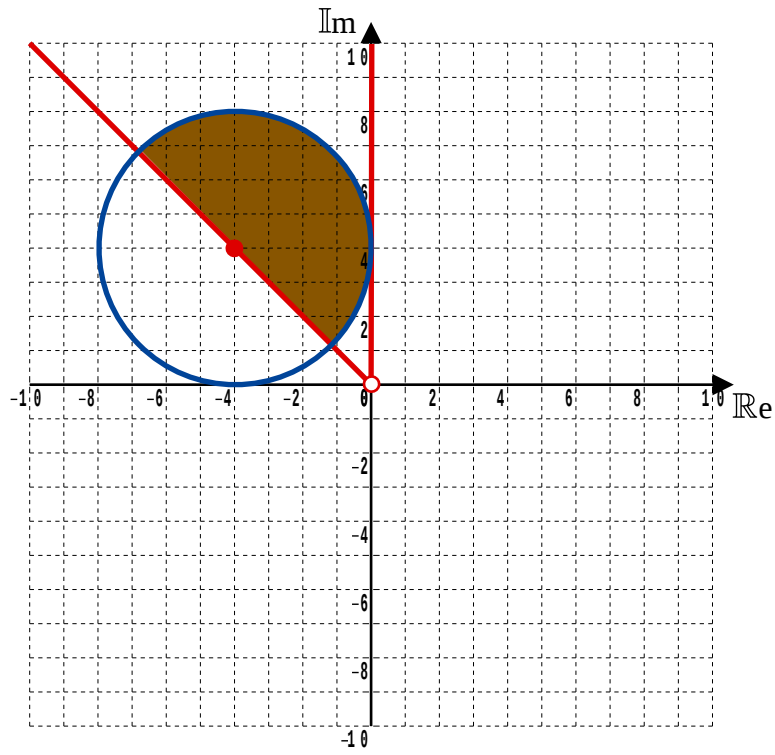
[2 marks]

Answer 2



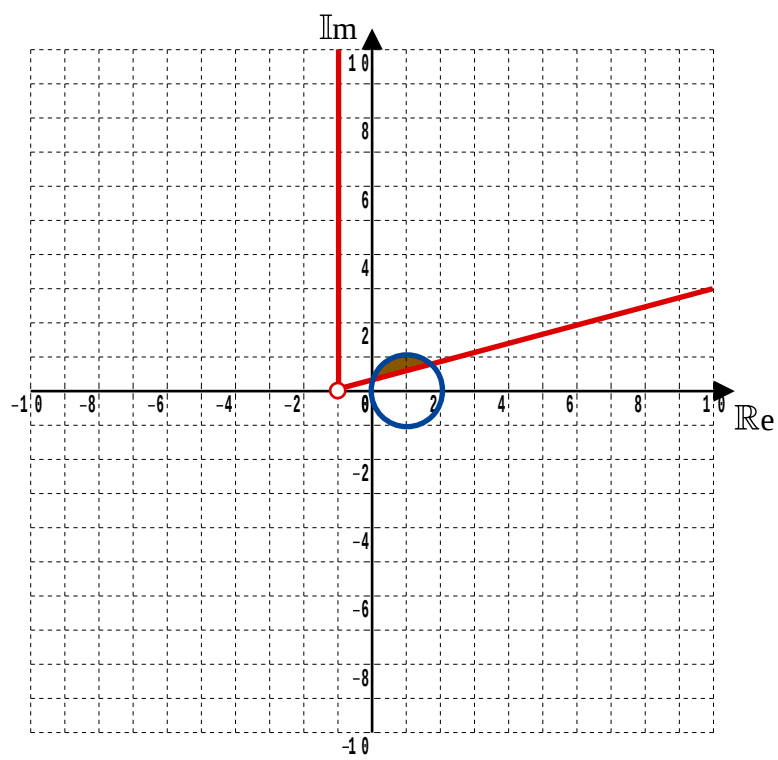
[2 marks]

Answer 3



[2 marks]

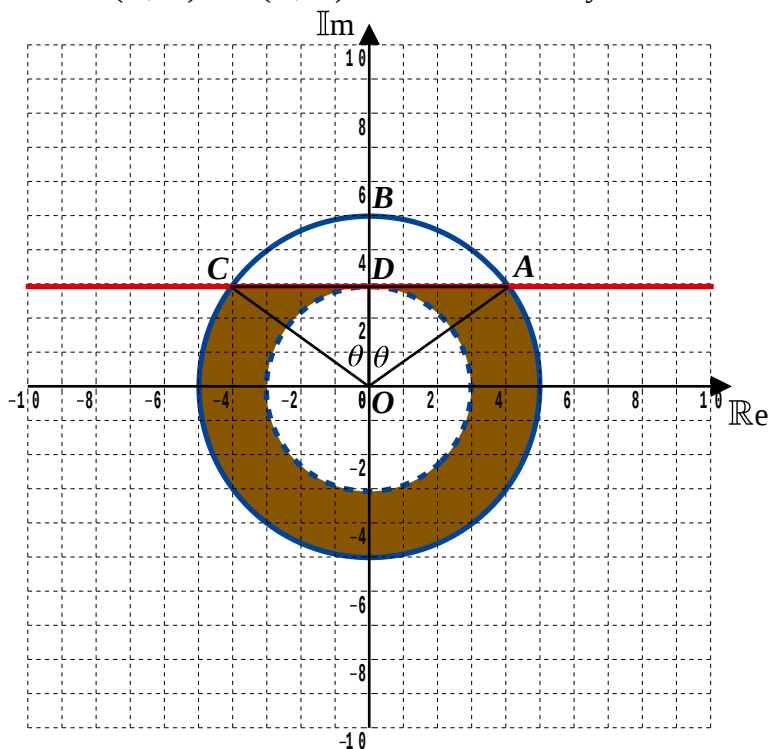
Answer 4



[4, 1 marks]

Answer 5

- (i) The $|z|$ is a circle centre $(0, 0)$: Write it as $|z - (0 + 0i)|$ if it helps.
 So this circle's radius varies between 3 and 5 giving an annulus (ring).
 $|z| = |z - 6i|$ could be written as $|z - (0 + 0i)| = |z - (0 + 6i)|$
 to make it clear that it is the perpendicular bisector of the line segment
 between $(0, 0)$ and $(0, 6)$ which is 'obviously' the line with equation $y = 3$.



[4 marks]

(ii)

$$|AD| = \sqrt{5^2 - 3^2}$$

$$= 4$$

$$\theta = \arctan\left(\frac{4}{3}\right)$$

$$= 0.9273$$

$$\text{Area Segment } ABCD = \text{Area Sector } OABC - \text{Area Triangle } OADC$$

$$= \frac{1}{2} \times 5^2 \times 2 \times 0.9273 - \frac{1}{2} \times 8 \times 3$$

$$= 23.18 - 12$$

$$= 11.18$$

$$\text{Shaded Area} = \text{Area Large Circle} - \text{Area Small Circle} - \text{Area Segment } ABCD$$

$$= \pi \times 5^2 - \pi \times 3^2 - 11.18$$

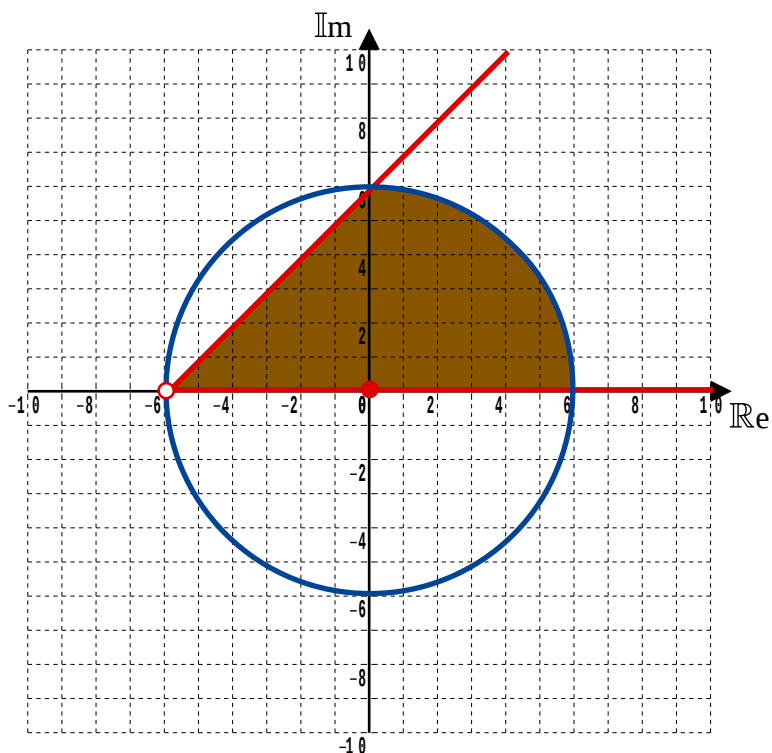
$$= 78.5 - 28.3 - 11.2$$

$$= 39.0$$

[3 marks]

Answer 6

(i)



[4 marks]

(ii)

Perimeter = Diameter + Quarter Circumference + Hypotenuse of Triangle

$$= 12 + \frac{1}{4} \times 2 \times \pi \times 6 + \sqrt{6^2 + 6^2}$$

$$= 12 + 3\pi + 6\sqrt{2}$$

[2 marks]

(iii)

Area shaded = Area Quarter Circle + Area Triangle

$$= \frac{1}{4} \times \pi \times 6^2 + \frac{1}{2} \times 6 \times 6$$

$$= 9\pi + 18$$

[2 marks]