Lesson 11

Further A-Level Pure Mathematics : Core 1

Complex Numbers I

11.1 Test

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 45

Question 1

In general a complex number z is often defined as being a number of the form $x + y\text{i}$.
In this definition which one of the following describes x and y ?

(A) Integers, $x, y \in \mathbb{Z}$

(B) Rational Numbers, $x, y \in \mathbb{Q}$

(C) Real Numbers, $x, y \in \mathbb{R}$

(D) Complex Numbers, $x, y \in \mathbb{C}$

[1 mark]

Question 2

Further A-Level Examination Question from May 2017, FP1, Q6

Given that 4 and $2\text{i} - 3$ are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where a and b are real constants,

(a) write down the third root of the equation

[1 mark]

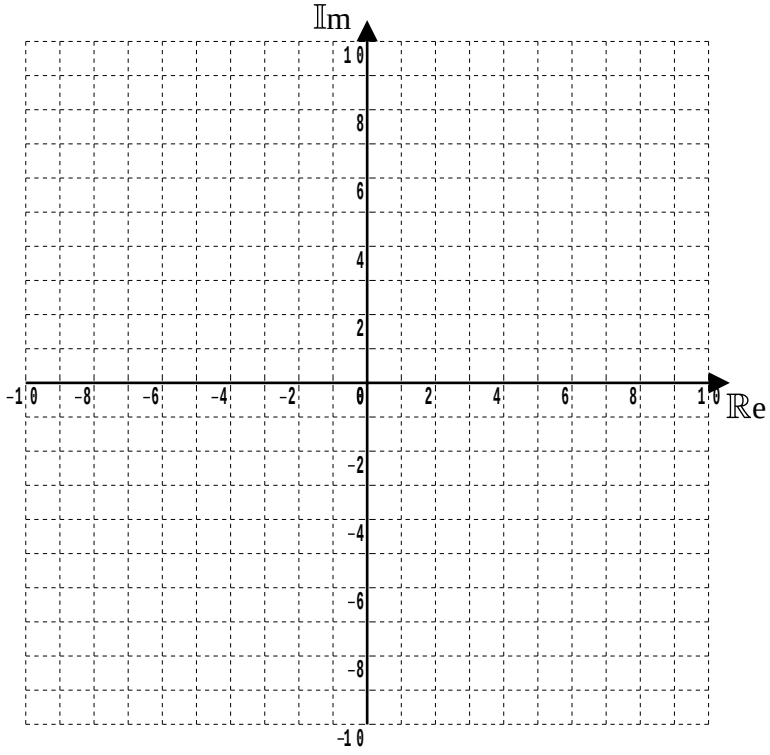
(b) Find the value of a and the value of b

[5 marks]

Question 3

(i) On the Argand diagram, shade in the region, *R*, specified by,

$$\{ z \in \mathbb{C} : 2.5 \leqslant |z| < 6.5 \} \cap \{ z \in \mathbb{C} : |z| \leqslant |z + 5i| \}$$



[5 marks]

(ii) Determine the area of the region *R*, correct to three significant figures.

[5 marks]

Question 4

Further A-Level Examination Question from January 2018, F1, Q5

(i) Given that

$$\frac{2z + 3}{z + 5 - 2\mathrm{i}} = 1 + \mathrm{i}$$

find z, giving your answer in the form $a + b\mathrm{i}$, where a and b are real constants.

[5 marks]

(ii) Given that

$$w = (3 + \lambda\mathrm{i})(2 + \mathrm{i})$$

where λ is a real constant, and that

$$|w| = 15$$

find the possible values of λ

[4 marks]

Question 5

Further A-Level Examination Question from June 2012, FP2, Q8

The point P represents a complex number z on an Argand diagram such that

$$|z - 6i| = 2|z - 3|$$

(a) Show that, as z varies, the locus of P is a circle, stating the radius and the coordinates of the centre of this circle

[6 marks]

The point Q represents a complex number z on an Argand diagram such that

$$arg\,(z - 6) = -\frac{3\pi}{4}$$

(b) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies

[4 marks]

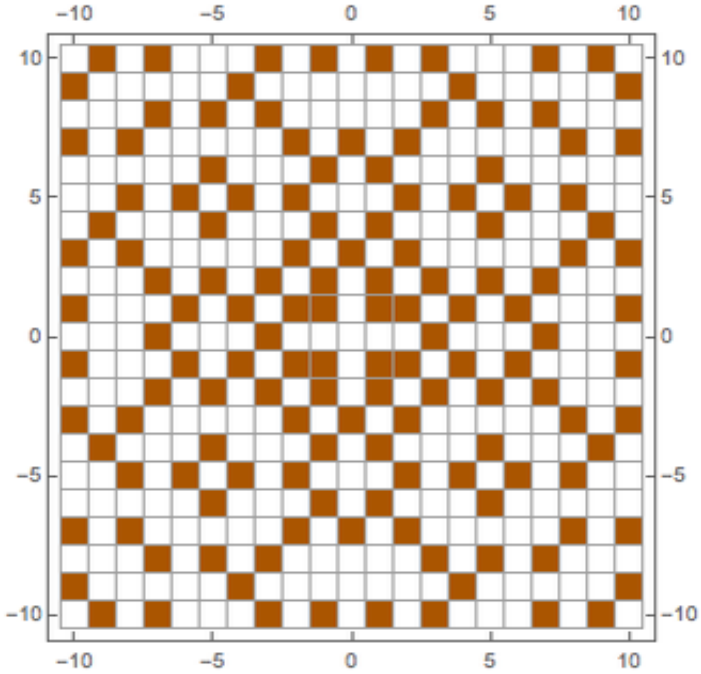
(c) Find the complex number for which both

$$|z - 6i| = 2|z - 3| \qquad \text{and} \qquad arg\,(z - 6) = -\frac{3\pi}{4}$$

[4 marks]

Question 6

A section of the grid of Gaussian Integers, $\mathbb{Z}[i]$, is shown.
In the grid, a dark square indicates a Gaussian Prime.



(i) Which one of the following four Gaussian Integers is a Gaussian Prime ?

$2i$ $\quad$ $2 + 3i$ $\quad$ $3 + 4i$ $\quad$ 5

[1 mark]

(ii) Find z such that $z^2 - 3 - 4i = 0$, $\quad$ $z \in \mathbb{Z}[i]$

[4 marks]

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