

1.5 Answers

Further A-Level Pure Mathematics : Core 1 Complex Numbers I

1.5.1 Solution to 1.3 Complex Number Arithmetic

$$\begin{aligned}\text{LHS} &= z^2 - 4z + 5 \\&= (2 + i)^2 - 4(2 + i) + 5 \\&= (2 + i)(2 + i) - 8 - 4i + 5 \\&= 4 + 2i + 2i + i^2 - 8 - 4i + 5 \\&= 4 + 2i + 2i - 1 - 8 - 4i + 5 \\&= 0 \\&= \text{RHS} \quad \square\end{aligned}$$

[3 marks]

1.5.2 Solutions to 1.4 Exercise

Answer 1

$$\begin{aligned}z^2 - 6z + 13 &= 0 \\[z^2 - 6z] + 13 &= 0 \\[(z - 3)^2 - 9] + 13 &= 0 \\(z - 3)^2 + 4 &= 0 \\(z - 3)^2 &= -4 \\z - 3 &= \pm \sqrt{(-1)(4)} \\z &= 3 \pm 2i\end{aligned}$$

[3 marks]

Answer 2

$$\begin{aligned}z^2 - 4z + 29 &= 0 \\[z^2 - 4z] + 29 &= 0 \\[(z - 2)^2 - 4] + 29 &= 0 \\(z - 2)^2 + 25 &= 0 \\(z - 2)^2 &= -25 \\z - 2 &= \pm \sqrt{(-1)(25)} \\z &= 2 \pm 5i\end{aligned}$$

[3 marks]

Answer 3

$$\begin{aligned}z^2 + 8z + 21 &= 0 \\[z^2 + 8z] + 21 &= 0 \\[(z + 4)^2 - 16] + 21 &= 0 \\(z + 4)^2 + 5 &= 0 \\(z + 4)^2 &= -5 \\z + 4 &= \pm \sqrt{(-1)(5)} \\z &= -4 \pm \sqrt{5} i\end{aligned}$$

[3 marks]

Answer 4

$$\begin{aligned}z^2 + 5z + 25 &= 0 \\z &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\&= \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 25}}{2 \times 1} \\&= \frac{-5 \pm \sqrt{25 - 100}}{2} \\&= \frac{-5}{2} \pm \frac{\sqrt{-75}}{2} \\&= -\frac{5}{2} \pm \frac{\sqrt{-1}\sqrt{75}}{2} \\&= -\frac{5}{2} \pm \frac{5\sqrt{3}}{2}i\end{aligned}$$

[3 marks]

Answer 5

$$\begin{aligned}\text{LHS} &= z^2 - 6z + 10 \\&= (3 + i)^2 - 6(3 + i) + 10 \\&= (3 + i)(3 + i) - 18 - 6i + 10 \\&= 9 + 3i + 3i + i^2 - 18 - 6i + 10 \\&= 9 + 3i + 3i - 1 - 18 - 6i + 10 \\&= 0 \\&= \text{RHS} \quad \square\end{aligned}$$

[3 marks]

Answer 6

$$\begin{aligned}
\text{LHS} &= f(z) \\
&= z^2 - 2z + 17 \\
&= (1 - 4i)^2 - 2(1 - 4i) + 17 \\
&= (1 - 4i)(1 - 4i) - 2 + 8i + 17 \\
&= 1 - 4i - 4i + 16i^2 - 2 + 8i + 17 \\
&= 1 - 4i - 4i - 16 - 2 + 8i + 17 \\
&= 0 \\
&= \text{RHS} \quad \square
\end{aligned}$$

[3 marks]**Answer 7****(i)**

$$\begin{aligned}
\text{LHS} &= z^4 + 4 \\
&= (1 + i)^4 + 4 \\
&= (1 + i)^2 (1 + i)^2 + 4 \\
&= (1 + 2i + i^2)(1 + 2i + i^2) + 4 \\
&= (1 + 2i - 1)(1 + 2i - 1) + 4 \\
&= (2i)(2i) + 4 \\
&= 4i^2 + 4 \\
&= 4(-1) + 4 \\
&= -4 + 4 \\
&= 0 \\
&= \text{RHS} \quad \square
\end{aligned}$$

[4 marks]**(ii)** Other three roots are,

$$1 - i, \quad -1 + i, \quad -1 - i$$

Whichever is guessed it then needs verification as in part (i)

[4 marks]**Answer 8**

$$\begin{aligned}
(a + bi)(a - bi) &= a^2 - abi + abi - b^2 i^2 \\
&= a^2 - b^2(-1) \\
&= a^2 + b^2 \text{ which is a real number} \quad \square
\end{aligned}$$

[3 marks]

Answer 9

Individual choice

[7 marks]**Answer 10**

$$\begin{aligned}
7z^2 - 3z + 3 &= 0 \\
[7z^2 - 3z] + 3 &= 0 \\
7\left[z^2 - \frac{3}{7}z\right] + 3 &= 0 \\
7\left[\left(z - \frac{3}{14}\right)^2 - \frac{9}{196}\right] + 3 &= 0 \\
7\left(z - \frac{3}{14}\right)^2 - \frac{9}{28} + 3 \times \frac{28}{28} &= 0 \\
7\left(z - \frac{3}{14}\right)^2 - \frac{9}{28} + \frac{84}{28} &= 0 \\
7\left(z - \frac{3}{14}\right)^2 &= -\frac{75}{28} \\
\left(z - \frac{3}{14}\right)^2 &= -\frac{75}{196} \\
\left(z - \frac{3}{14}\right) &= \pm \frac{\sqrt{-75}}{14} \\
z &= \frac{3}{14} \pm \frac{\sqrt{-1 \times 3 \times 25}}{14} \\
&= \frac{3}{14} \pm \frac{5\sqrt{3}}{14} i
\end{aligned}$$

[4 marks]

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