1.5 Answers

Further A-Level Pure Mathematics : Core 1
Complex Numbers I

1.5.1 Solution to 1.3 Complex Number Arithmetic

$$\begin{aligned}
\text{LHS} &= z^2 - 4z + 5 \\
&= (2 + i)^2 - 4(2 + i) + 5 \\
&= (2 + i)(2 + i) - 8 - 4i + 5 \\
&= 4 + 2i + 2i + i^2 - 8 - 4i + 5 \\
&= 4 + 2i + 2i - 1 - 8 - 4i + 5 \\
&= 0 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

1.5.2 Solutions to 1.4 Exercise

Answer 1

$$\begin{aligned}
z^2 - 6z + 13 &= 0 \\
\left[z^2 - 6z \right] + 13 &= 0 \\
\left[(z - 3)^2 - 9 \right] + 13 &= 0 \\
(z - 3)^2 + 4 &= 0 \\
(z - 3)^2 &= -4 \\
z - 3 &= \pm\sqrt{(-1)(4)} \\
z &= 3 \pm 2i
\end{aligned}$$

[3 marks]

Answer 2

$$\begin{aligned}
z^2 - 4z + 29 &= 0 \\
\left[z^2 - 4z \right] + 29 &= 0 \\
\left[(z - 2)^2 - 4 \right] + 29 &= 0 \\
(z - 2)^2 + 25 &= 0 \\
(z - 2)^2 &= -25 \\
z - 2 &= \pm\sqrt{(-1)(25)} \\
z &= 2 \pm 5i
\end{aligned}$$

[3 marks]

Answer 3

$$z^2 + 8z + 21 = 0$$
$$\left[z^2 + 8z \right] + 21 = 0$$
$$\left[(z + 4)^2 - 16 \right] + 21 = 0$$
$$(z + 4)^2 + 5 = 0$$
$$(z + 4)^2 = -5$$
$$z + 4 = \pm\sqrt{(-1)(5)}$$
$$z = -4 \pm \sqrt{5}\,\mathrm{i}$$

[3 marks]

Answer 4

$$z^2 + 5z + 25 = 0$$
$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 25}}{2 \times 1}$$
$$= \frac{-5 \pm \sqrt{25 - 100}}{2}$$
$$= \frac{-5}{2} \pm \frac{\sqrt{-75}}{2}$$
$$= -\frac{5}{2} \pm \frac{\sqrt{-1}\sqrt{75}}{2}$$
$$= -\frac{5}{2} \pm \frac{5\sqrt{3}}{2}\mathrm{i}$$

[3 marks]

Answer 5

$$\text{LHS} = z^2 - 6z + 10$$
$$= (3 + \mathrm{i})^2 - 6(3 + \mathrm{i}) + 10$$
$$= (3 + \mathrm{i})(3 + \mathrm{i}) - 18 - 6\mathrm{i} + 10$$
$$= 9 + 3\mathrm{i} + 3\mathrm{i} + \mathrm{i}^2 - 18 - 6\mathrm{i} + 10$$
$$= 9 + 3\mathrm{i} + 3\mathrm{i} - 1 - 18 - 6\mathrm{i} + 10$$
$$= 0$$
$$= \text{RHS} \qquad \square$$

[3 marks]

Answer 6

$$\begin{aligned}
\text{LHS} &= f(z) \\
&= z^2 - 2z + 17 \\
&= (1 - 4\text{i})^2 - 2(1 - 4\text{i}) + 17 \\
&= (1 - 4\text{i})(1 - 4\text{i}) - 2 + 8\text{i} + 17 \\
&= 1 - 4\text{i} - 4\text{i} + 16\,\text{i}^2 - 2 + 8\text{i} + 17 \\
&= 1 - 4\text{i} - 4\text{i} - 16 - 2 + 8\text{i} + 17 \\
&= 0 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

Answer 7

(i)

$$\begin{aligned}
\text{LHS} &= z^4 + 4 \\
&= (1 + \text{i})^4 + 4 \\
&= (1 + \text{i})^2 (1 + \text{i})^2 + 4 \\
&= \left(1 + 2\text{i} + \text{i}^2\right)\left(1 + 2\text{i} + \text{i}^2\right) + 4 \\
&= \left(1 + 2\text{i} - 1\right)\left(1 + 2\text{i} - 1\right) + 4 \\
&= (2\text{i})(2\text{i}) + 4 \\
&= 4\,\text{i}^2 + 4 \\
&= 4(-1) + 4 \\
&= -4 + 4 \\
&= 0 \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[4 marks]

(ii) Other three roots are,

$$1 - \text{i}, \qquad -1 + \text{i}, \qquad -1 - \text{i}$$

Whichever is guessed it then needs verification as in part (i)

[4 marks]

Answer 8

$$\begin{aligned}
(a + b\text{i})(a - b\text{i}) &= a^2 - ab\text{i} + ab\text{i} - b^2\,\text{i}^2 \\
&= a^2 - b^2(-1) \\
&= a^2 + b^2 \text{ which is a real number} \qquad \square
\end{aligned}$$

[3 marks]

Answer 9

Individual choice

[7 marks]

Answer 10

$$7z^2 - 3z + 3 = 0$$

$$\left[7z^2 - 3z \right] + 3 = 0$$

$$7\left[z^2 - \frac{3}{7} z \right] + 3 = 0$$

$$7\left[\left(z - \frac{3}{14} \right)^2 - \frac{9}{196} \right] + 3 = 0$$

$$7\left(z - \frac{3}{14} \right)^2 - \frac{9}{28} + 3 \times \frac{28}{28} = 0$$

$$7\left(z - \frac{3}{14} \right)^2 - \frac{9}{28} + \frac{84}{28} = 0$$

$$7\left(z - \frac{3}{14} \right)^2 = -\frac{75}{28}$$

$$\left(z - \frac{3}{14} \right)^2 = -\frac{75}{196}$$

$$\left(z - \frac{3}{14} \right) = \pm\frac{\sqrt{-75}}{14}$$

$$z = \frac{3}{14} \pm \frac{\sqrt{-1 \times 3 \times 25}}{14}$$

$$= \frac{3}{14} \pm \frac{5\sqrt{3}}{14}\,\mathrm{i}$$

[4 marks]