8.5 Answers

8.5.1 Solution to 8.2 The Perpendicular Bisector

To find the perpendicular bisector of the line segment with endpoints $(-2,\ 1)$ and we can first find the midpoint between those endpoints,

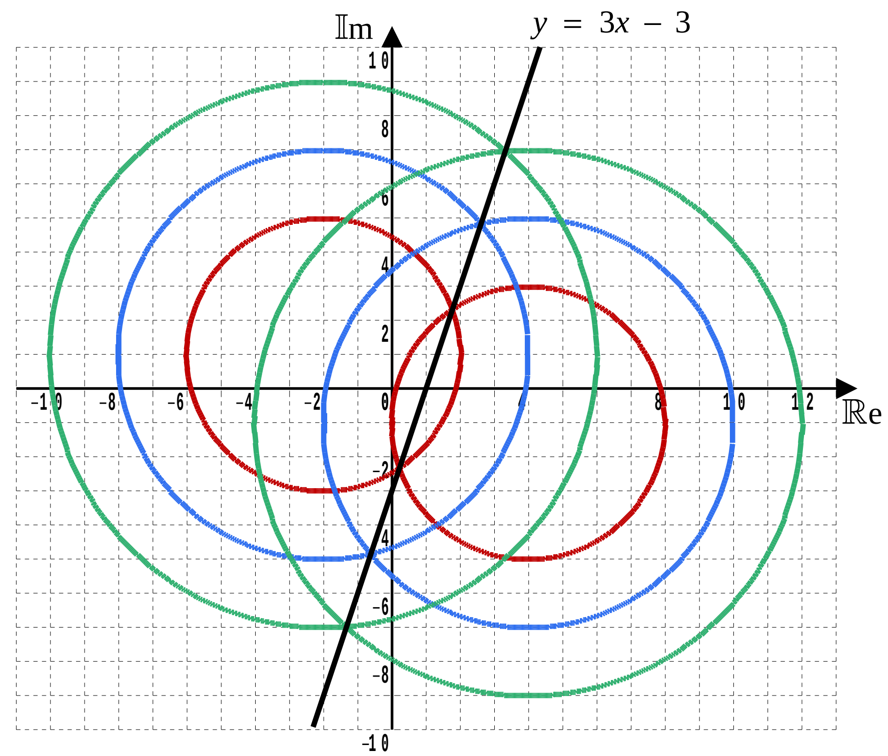
$$(\text{mean } x,\ \text{mean } y) = \left(\frac{-2 + 4}{2},\ \frac{1 + (-1)}{2}\right)$$
$$= (1,\ 0)$$

The gradient of the line between the two circle centres will be,

$$m = \frac{\Delta Y}{\Delta X}$$
$$= \frac{(-1) - (1)}{(4) - (-2)}$$
$$= \frac{-2}{6}$$
$$= -\frac{1}{3}$$

The line perpendicular to this will have a gradient that is “the sign changed reciprocal”. Thus we're after a line with gradient 3 through (1, 0),

$$y = mx + c$$
$$0 = 3 \times 1 + c \Rightarrow c = -3$$
$$\therefore\ y = 3x - 3$$



[3 marks]

8.5.2 Solutions to 8.4 Exercise

Answer 1

(a) Centre is (10, 5)

[1 mark]

(b) Centre is $(-6,\ 1)$

[1 mark]

(c)

$$\text{Midpoint } \left(\frac{10 + (-6)}{2}, \frac{5 + 1}{2} \right) = (2,\ 3)$$

$$m = \frac{\Delta Y}{\Delta X}$$

$$= \frac{5 - 1}{10 - (-6)}$$

$$= \frac{4}{16}$$

$$= \frac{1}{4} \qquad \therefore m_{PERP} = -4$$

$$y = mx + c$$

$$3 = -4 \times 2 + c \Rightarrow c = 11$$

Cartesian Equation of perpendicular bisector is $y = -4x + 11$

[3 marks]

(d) The gradient of the shortest line will be 0.25 and it has to go through (0, 0) so it will have equation,

$$y = 0.25x$$

To get the point of intersection with the perpendicular bisector,

$$0.25x = -4x + 11$$

$$x = -16x + 44$$

$$17x = 44$$

$$x = \frac{44}{17} \qquad \therefore y = \frac{11}{17}$$

$$\text{Minimum Distance} = \sqrt{\left(\frac{44}{17}\right)^2 + \left(\frac{11}{17}\right)^2}$$

$$= \frac{11\sqrt{17}}{17} \qquad (2.66789...)$$

[3 marks]

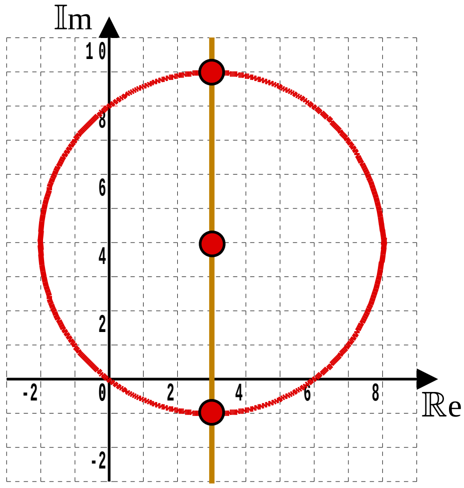
Answer 2

(a) $|z - 6|$ can be thought of as a circle of unspecified radius, but centre (6, 0)

$|z|$ can be thought of as a circle of unspecified radius, but centre (0, 0)

The locus will be the perpendicular bisector of the line segment between these two point, which will be $x = 3$

(b) $|w - 3 - 4i| = 5$ can be thought of an a circle of radius 5, centre (3, 4)



(c) The complex numbers on both loci are,

$$z_1 = w_1 = 3 - i \qquad z_2 = w_2 = 3 + 9i$$

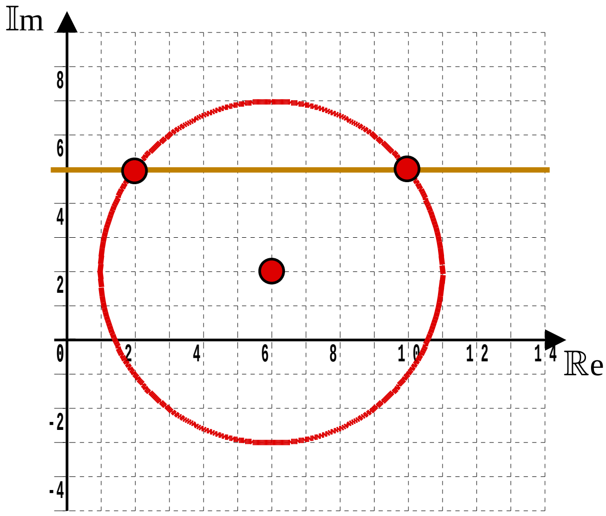
[2, 2, 2 marks]

Answer 3

(a) $|z - 2i| = |z - 8i|$ will have locus that is the perpendicular bisector of the line segment between (0, 2) and (0, 8) which are the centres of the circles described by $|z - 2i| = \lambda$ and $|z - 8i| = \lambda$ where λ is a variable radius.

Without calculation, this will be obviously be the line $y = 5$

(b) $|z - 6 - 2i| = 5$ is a circle radius 5, centre (6, 2) on the Argand diagram.



(c) The complex numbers on both loci are

$$z_1 = 2 + 5i \qquad \text{and} \qquad z_2 = 10 + 5i$$

[2, 3, 2 marks]

Answer 4

For P a cautious approach is required,

$$|z+1| = |2z-1|$$
$$|x+y\text{i}+1| = |2(x+y\text{i})-1|$$
$$|(x+1)+y\text{i}| = |(2x-1)+2y\text{i}|$$

Take the modulus of each side, and square to remove the square roots

$$(x+1)^2 + y^2 = (2x-1)^2 + (2y)^2$$
$$x^2 + 2x + 1 + y^2 = 4x^2 - 4x + 1 + 4y^2$$
$$3x^2 - 6x + 3y^2 = 0$$

Divide through by 3

$$\left[x^2 - 2x\right] + y^2 = 0$$
$$\left[(x-1)^2 - 1\right] + y^2 = 0$$
$$(x-1)^2 + y^2 = 1$$

This is the equation of a circle, centre (1, 0) and radius 1

For Q it's the perpendicular bisector of the line segment between (0, 0) and $(1, -1)$ that is required which will be the line $y = x - 1$

Solving by substitution,

$$(x-1)^2 + y^2 = 1$$
$$(x-1)^2 + (x-1)^2 = 1$$
$$2(x-1)^2 = 1$$
$$(x-1)^2 = \frac{1}{2}$$
$$x - 1 = \pm\frac{1}{\sqrt{2}}$$
$$x - 1 = \pm\frac{\sqrt{2}}{2}$$
$$x = 1 \pm \frac{\sqrt{2}}{2}$$
$$x = 1 - \frac{\sqrt{2}}{2} \quad \text{or} \quad x = 1 + \frac{\sqrt{2}}{2}$$

Points of intersection are

$$\left(1 - \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \quad \text{or} \quad \left(1 + \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

[7 marks]

Answer 5

(a)

$$|z + 6| = 2|z - 6|$$

$$|x + yi + 6| = 2|x + yi - 6|$$

$$|(x + 6) + yi| = 2|(x - 6) + yi|$$

Take the modulus of each side, and square to remove the square roots

$$(x + 6)^2 + y^2 = 4\left[(x - 6)^2 + y^2\right]$$

$$x^2 + 12x + 36 + y^2 = 4\left[x^2 - 12x + 36 + y^2\right]$$

$$x^2 + 12x + 36 + y^2 = 4x^2 - 48x + 144 + 4y^2$$

$$3x^2 - 60x + 3y^2 + 108 = 0$$

Divide through by 3

$$x^2 + y^2 - 20x + 36 = 0 \qquad \square$$

[2 marks]

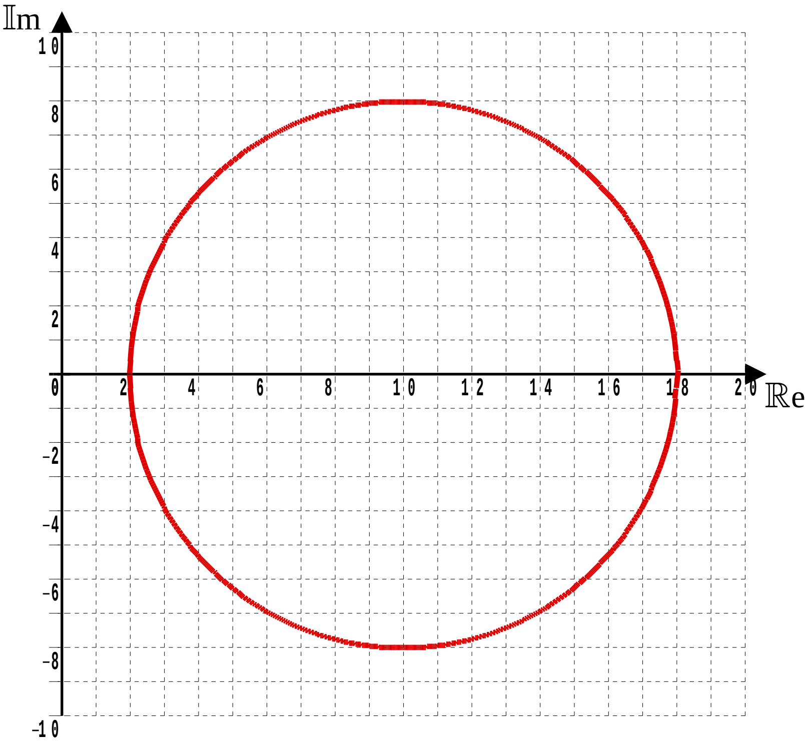
(b)

$$\left[x^2 - 20x\right] + y^2 + 36 = 0$$

$$\left[(x - 10)^2 - 100\right] + y^2 + 36 = 0$$

$$(x - 10)^2 + y^2 = 64$$

Which is recognizable as a circle, centre (10, 0) and radius 8



[2 marks]