

8.5 Answers

Further A-Level Pure Mathematics : Core 1

Complex Numbers I

8.5.1 Solution to 8.2 The Perpendicular Bisector

To find the perpendicular bisector of the line segment with endpoints $(-2, 1)$ and we can first find the midpoint between those endpoints,

$$\begin{aligned}(\text{mean } x, \text{ mean } y) &= \left(\frac{-2 + 4}{2}, \frac{1 + (-1)}{2} \right) \\&= (1, 0)\end{aligned}$$

The gradient of the line between the two circle centres will be,

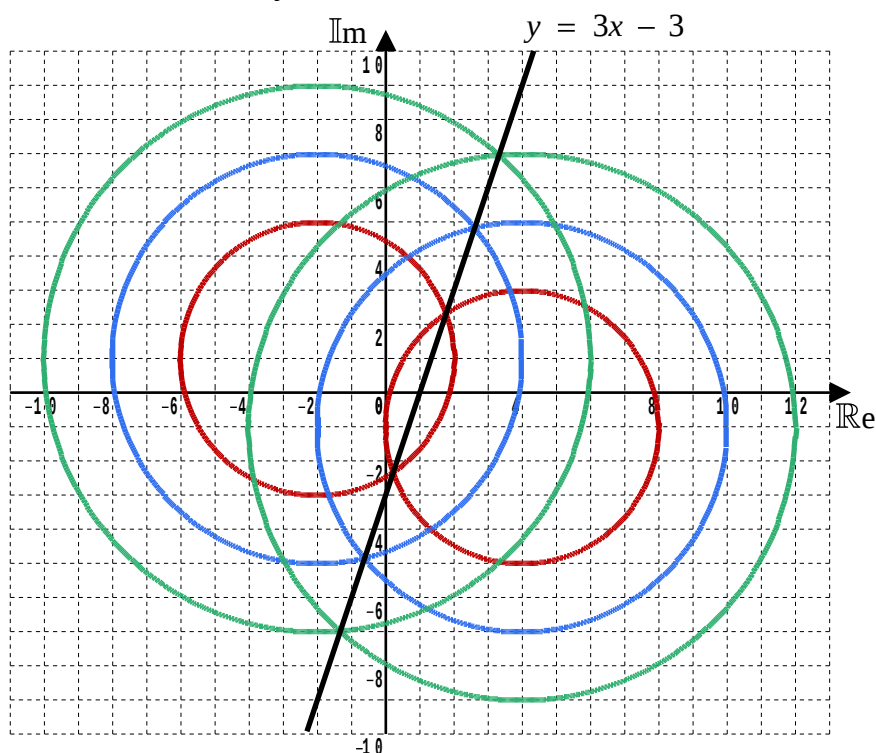
$$\begin{aligned}m &= \frac{\Delta Y}{\Delta X} \\&= \frac{(-1) - (1)}{(4) - (-2)} \\&= \frac{-2}{6} \\&= -\frac{1}{3}\end{aligned}$$

The line perpendicular to this will have a gradient that is “the sign changed reciprocal”. Thus we're after a line with gradient 3 through $(1, 0)$,

$$y = mx + c$$

$$0 = 3 \times 1 + c \Rightarrow c = -3$$

$$\therefore y = 3x - 3$$



[3 marks]

8.5.2 Solutions to 8.4 Exercise

Answer 1

(a) Centre is (10, 5)

[1 mark]

(b) Centre is (−6, 1)

[1 mark]

(c)

$$\text{Midpoint} \left(\frac{10 + (-6)}{2}, \frac{5 + 1}{2} \right) = (2, 3)$$

$$\begin{aligned} m &= \frac{\Delta Y}{\Delta X} \\ &= \frac{5 - 1}{10 - (-6)} \\ &= \frac{4}{16} \\ &= \frac{1}{4} \quad \therefore m_{\text{PERP}} = -4 \end{aligned}$$

$$y = mx + c$$

$$3 = -4 \times 2 + c \Rightarrow c = 11$$

Cartesian Equation of perpendicular bisector is $y = -4x + 11$

[3 marks]

(d) The gradient of the shortest line will be 0.25 and it has to go through (0, 0) so it will have equation,

$$y = 0.25x$$

To get the point of intersection with the perpendicular bisector,

$$0.25x = -4x + 11$$

$$x = -16x + 44$$

$$17x = 44$$

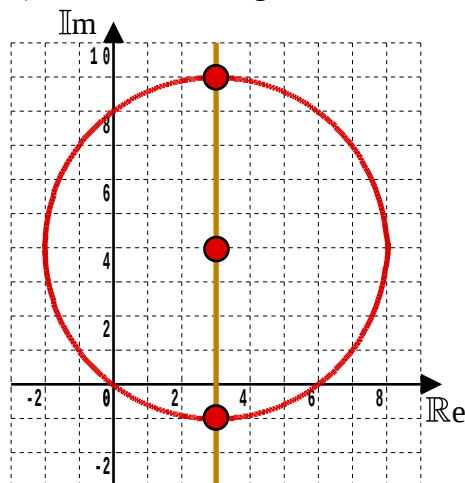
$$x = \frac{44}{17} \quad \therefore y = \frac{11}{17}$$

$$\begin{aligned} \text{Minimum Distance} &= \sqrt{\left(\frac{44}{17}\right)^2 + \left(\frac{11}{17}\right)^2} \\ &= \frac{11\sqrt{17}}{17} \quad (2.66789...) \end{aligned}$$

[3 marks]

Answer 2

- (a) $|z - 6|$ can be thought of as a circle of unspecified radius, but centre $(6, 0)$
 $|z|$ can be thought of as a circle of unspecified radius, but centre $(0, 0)$
 The locus will be the perpendicular bisector of the line segment between these two points, which will be $x = 3$
- (b) $|w - 3 - 4i| = 5$ can be thought of as a circle of radius 5, centre $(3, 4)$



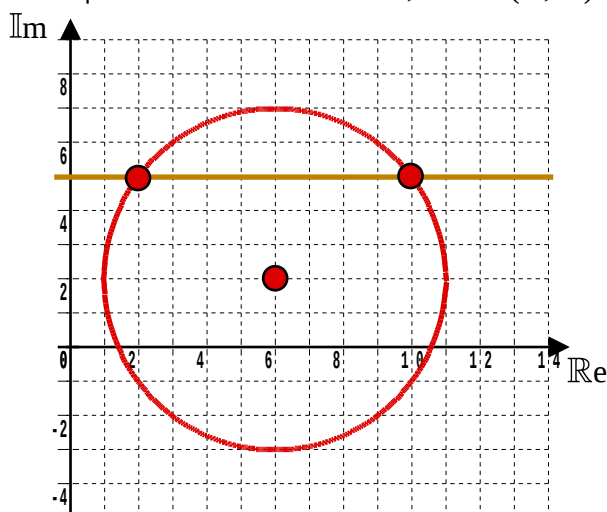
- (c) The complex numbers on both loci are,

$$z_1 = w_1 = 3 - i \quad z_2 = w_2 = 3 + 9i$$

[2, 2, 2 marks]

Answer 3

- (a) $|z - 2i| = |z - 8i|$ will have locus that is the perpendicular bisector of the line segment between $(0, 2)$ and $(0, 8)$ which are the centres of the circles described by $|z - 2i| = \lambda$ and $|z - 8i| = \lambda$ where λ is a variable radius. Without calculation, this will obviously be the line $y = 5$
- (b) $|z - 6 - 2i| = 5$ is a circle radius 5, centre $(6, 2)$ on the Argand diagram.



- (c) The complex numbers on both loci are

$$z_1 = 2 + 5i \quad \text{and} \quad z_2 = 10 + 5i$$

[2, 3, 2 marks]

Answer 4

For P a cautious approach is required,

$$|z + 1| = |2z - 1|$$

$$|x + yi + 1| = |2(x + yi) - 1|$$

$$|(x + 1) + yi| = |(2x - 1) + 2yi|$$

Take the modulus of each side, and square to remove the square roots

$$(x + 1)^2 + y^2 = (2x - 1)^2 + (2y)^2$$

$$x^2 + 2x + 1 + y^2 = 4x^2 - 4x + 1 + 4y^2$$

$$3x^2 - 6x + 3y^2 = 0$$

Divide through by 3

$$[x^2 - 2x] + y^2 = 0$$

$$[(x - 1)^2 - 1] + y^2 = 0$$

$$(x - 1)^2 + y^2 = 1$$

This is the equation of a circle, centre $(1, 0)$ and radius 1

For Q it's the perpendicular bisector of the line segment between $(0, 0)$ and $(1, -1)$ that is required which will be the line $y = x - 1$

Solving by substitution,

$$(x - 1)^2 + y^2 = 1$$

$$(x - 1)^2 + (x - 1)^2 = 1$$

$$2(x - 1)^2 = 1$$

$$(x - 1)^2 = \frac{1}{2}$$

$$x - 1 = \pm \frac{1}{\sqrt{2}}$$

$$x - 1 = \pm \frac{\sqrt{2}}{2}$$

$$x = 1 \pm \frac{\sqrt{2}}{2}$$

$$x = 1 - \frac{\sqrt{2}}{2} \quad \text{or} \quad x = 1 + \frac{\sqrt{2}}{2}$$

Points of intersection are

$$\left(1 - \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \quad \text{or} \quad \left(1 + \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

[7 marks]

Answer 5

$$\begin{aligned}
 \text{(a)} \quad |z + 6| &= 2|z - 6| \\
 |x + yi + 6| &= 2|x + yi - 6| \\
 |(x + 6) + yi| &= 2|(x - 6) + yi|
 \end{aligned}$$

Take the modulus of each side, and square to remove the square roots

$$\begin{aligned}
 (x + 6)^2 + y^2 &= 4[(x - 6)^2 + y^2] \\
 x^2 + 12x + 36 + y^2 &= 4[x^2 - 12x + 36 + y^2] \\
 x^2 + 12x + 36 + y^2 &= 4x^2 - 48x + 144 + 4y^2 \\
 3x^2 - 60x + 3y^2 + 108 &= 0
 \end{aligned}$$

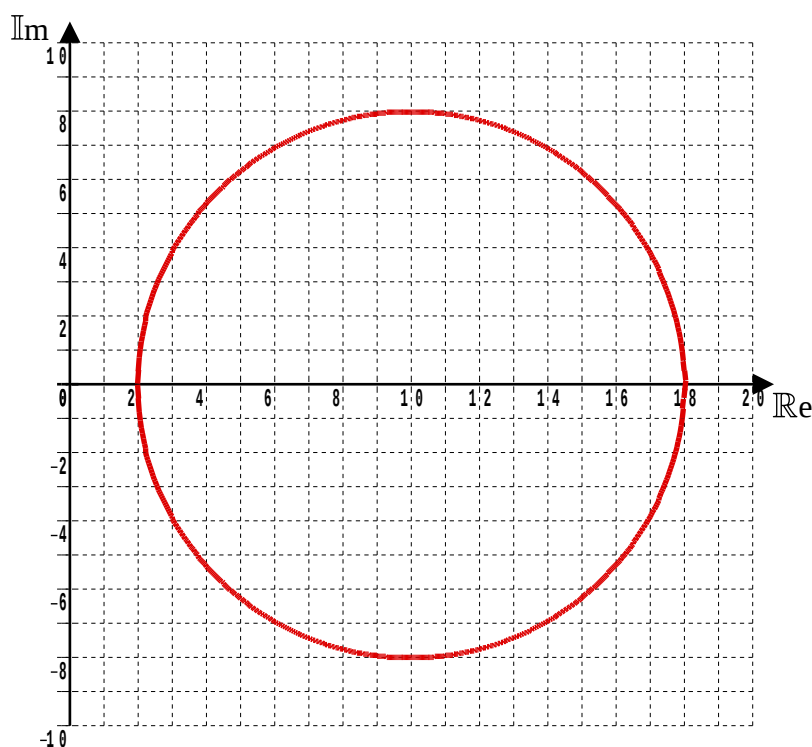
Divide through by 3

$$x^2 + y^2 - 20x + 36 = 0 \quad \square$$

[2 marks]

$$\begin{aligned}
 \text{(b)} \quad [x^2 - 20x] + y^2 + 36 &= 0 \\
 [(x - 10)^2 - 100] + y^2 + 36 &= 0 \\
 (x - 10)^2 + y^2 &= 64
 \end{aligned}$$

Which is recognizable as a circle, centre (10, 0) and radius 8



[2 marks]

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