Year 1 Pure Mathematics Examination Revision
Health Check N° 1

Answer 1

(i)

$$x^2 - 8x - 13 = \left[x^2 - 8x \right] - 13$$
$$= \left[(x - 4)^2 - 16 \right] - 13$$
$$= (x - 4)^2 - 29$$

[2 marks]

(ii) The minimum value of the function is – 29

[1 mark]

(iii) The minimum value occurs when $x = 4$

[1 mark]

Answer 2

(i)

$$\begin{aligned}(1 - 2x)^8 = \;& 1 \times (1)^8 \times (-2x)^0 \\ &+ 8 \times (1)^7 \times (-2x)^1 \\ &+ 28 \times (1)^6 \times (-2x)^2 \\ &+ 56 \times (1)^5 \times (-2x)^3 \\ &+ \ldots \\ = \;& 1 - 16x + 112x^2 - 448x^3 \quad \left(\text{Up to the term in } x^3\right)\end{aligned}$$

[3 marks]

(ii) We require $1 - 2x = 0.98$

$$2x = 0.02$$
$$x = 0.01$$

$$\begin{aligned}\therefore\; 0.98^8 &\approx 1 - 16 \times 0.01 + 112 \times 0.01^2 - 448 \times 0.01^3 \\ &\approx 1 - 0.16 + 0.0112 - 0.000448 \\ &\approx 0.850752 \\ &\approx 0.8508 \text{ (4 decimal places requested)}\end{aligned}$$

[2 marks]

Answer 3

(i)

$$\sin 2x = 0.5$$

$$2x = 30,\ 150,\ 390,\ 510,\ \ldots$$

$$x = 15,\ 75,\ 195,\ 255$$

[4 marks]

(ii) Using the identity $\cos^2 x + \sin^2 x = 1$,

$$\sin^2 x = \cos x - 1$$

$$1 - \cos^2 x = \cos x - 1$$

$$\cos^2 x + \cos x - 2 = 0$$

$$(\cos x + 2)(\cos x - 1) = 0$$

Either $\cos x = -2$ which has no solutions

Or $\cos x = 1$ which has solutions

$x = 0, \pm 360$ over the domain specified

[4 marks]

Answer 4

(i)

$$y = \frac{x^2 + 2x + 3}{x}$$

$$= \frac{x^2}{x} + \frac{2x}{x} + \frac{3}{x}$$

$$= x + 2 + 3x^{-1}$$

$$\frac{dy}{dx} = 1 - 3x^{-2}$$

$$= 1 - \frac{3}{x^2}$$

[4 marks]

(ii)

$$\int \frac{4x - 9}{\sqrt{x}}\,dx = \int \frac{4x}{\sqrt{x}} - \frac{9}{\sqrt{x}}\,dx$$

$$= \int 4x^{\frac{1}{2}} - 9x^{-\frac{1}{2}}\,dx$$

$$= \left[\frac{4 \times 2x^{\frac{3}{2}}}{3} - \frac{9 \times 2x^{\frac{1}{2}}}{1} \right] + c$$

$$= \frac{8}{3}x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + c$$

$$= \frac{2}{3}\sqrt{x}\,(4x - 27) + c$$

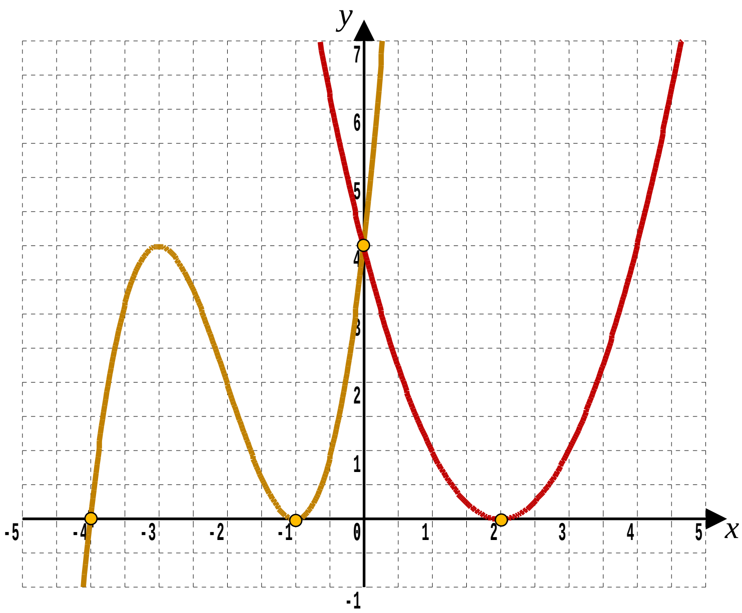
[4 marks]

Answer 5

(i) $y = (x - 2)^2$ is plotted in red

(ii) $y = (x + 4)(x + 1)^2$ is plotted in gold

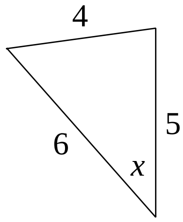
The examiner is looking for the correct overall shape and points where the curves cross the coordinate axes.



[5 marks]

Answer 6

(i) The smallest angle is opposite the shortest side.



$$\text{By The Cosine Rule: } \cos x = \frac{6^2 + 5^2 - 4^2}{2 \times 6 \times 5}$$

$$= \frac{3}{4}$$

$$\therefore x = \arccos\left(\frac{3}{4}\right)$$

$$= 41.4°$$

[3 marks]

(ii)

$$Area\,\Delta = \frac{1}{2}\, 6 \times 5 \times \sin 41.4°$$

$$= 9.92 \text{ cm}^2$$

[3 marks]

Answer 7

$$x - y = 2 \Rightarrow y = x - 2$$

$$x^2 + y^2 = 100$$

$$x^2 + (x - 2)^2 = 100$$

$$x^2 + x^2 - 4x + 4 = 100$$

$$2x^2 - 4x - 96 = 0$$

divide through by 2

$$x^2 - 2x - 48 = 0$$

$$(x - 8)(x + 6) = 0$$

Either $x = 8$ in which case $y = 6$

Or $x = -6$ in which case $y = -8$

That is, the solutions are $(8, 6)$ or $(-6, -8)$

[4 marks]

Answer 8

(i)

$$4x^2 + 3 < 39$$

$$4x^2 < 36$$

$$x^2 < 9$$

$$-3 < x < 3$$

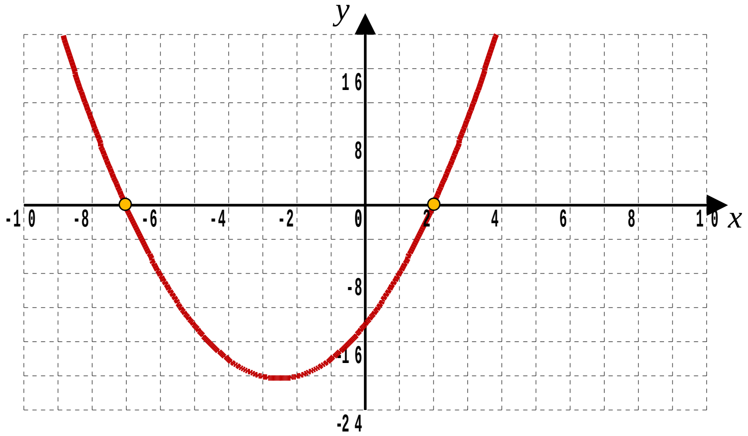
[3 marks]

(ii)

$$x^2 + 5x > 14$$

$$x^2 + 5x - 14 > 0$$

$$(x + 7)(x - 2) > 0$$



Solutions are $x < -7$ or $x > 2$

[3 marks]

Answer 9

(i)

$$x^2 + y^2 - 6x + 4y - 3 = 0$$

$$\left[x^2 - 6x \right] + \left[y^2 + 4y \right] - 3 = 0$$

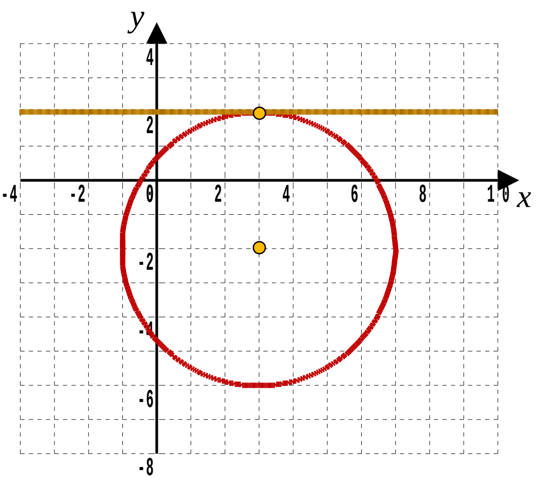
$$\left[(x - 3)^2 - 9 \right] + \left[(y + 2)^2 - 4 \right] - 3 = 0$$

$$(x - 3)^2 + (y + 2)^2 = 16$$

which is a circle with centre $(3, -2)$ and radius 4

[3 marks]

(ii) If you visualise what's going on the answer becomes obvious !



The tangent is $y = 2$

[4 marks]

Answer 10

(i)

$$2x + 3y = 13$$

$$3y = -2x + 13$$

$$y = -\frac{2}{3}x + \frac{13}{3}$$

$\therefore$ L_2 has gradient $-\dfrac{2}{3}$ and passes through $(1,\ 5)$

$$y = mx + c$$

$$5 = -\frac{2}{3} \times 1 + c$$

$$c = \frac{17}{3}$$

$$y = -\frac{2}{3}x + \frac{17}{3} \quad \text{or} \quad 3y + 2x = 17$$

[2 marks]

(ii) L_3 has gradient $\frac{3}{2}$ and passes through $(1,\ 5)$

$$y = mx + c$$

$$5 = \frac{3}{2} \times 1 + c$$

$$c = \frac{7}{2}$$

$$y = \frac{3}{2}x + \frac{7}{2} \quad \text{or} \quad 2y - 3x = 7$$

[2 marks]

Answer 11

(i)

$$y = x^3 - 3x^2 - 9x + 15$$

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

At turning points $\frac{dy}{dx} = 0$

$$\therefore\ 3x^2 - 6x - 9 = 0$$

divide through by 3

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

Either $x = 3$ in which case $y = -12$

Or $x = -1$ in which case $y = 20$

That is, the turning points are $(-1,\ 20)$, $(3, -12)$

[4 marks]

(ii) From a knowledge of the shape of a cubic curve

$(-1,\ 20)$ is a maximum and $(3, -12)$ is a minimum

Alternatively,

$$\frac{d^2y}{dx^2} = 6x - 6$$

When $x = -1$, $\frac{d^2y}{dx^2} = -12$, i.e. negative $\Rightarrow$ $(-1,\ 20)$ is a maximum

When $x = 3$, $\frac{d^2y}{dx^2} = 12$, i.e. positive $\Rightarrow$ $(3, -12)$ is a minimum

[2 marks]

Answer 12

(i)

$$2^{2x} - 3 \times 2^{x} + 2 = 0$$

$$\left(2^{x}\right)^{2} - 3\left(2^{x}\right) + 2 = 0 \qquad \text{“Quadratic in disguise”}$$

$$\left(2^{x} - 2\right)\left(2^{x} - 1\right) = 0$$

Either $2^{x} = 2$ in which case $x = 1$

Or $2^{x} = 1$ in which case $x = 0$

[3 marks]

(ii)

$$2\,log_3(x + 2) - log_3 x = 2$$

$$log_3(x + 2)^2 - log_3 x = 2$$

$$log_3\left(\frac{(x + 2)^2}{x}\right) = 2$$

$$\therefore \quad \frac{(x + 2)^2}{x} = 3^2$$

$$x^2 + 4x + 4 = 9x$$

$$x^2 - 5x + 4 = 0$$

$$(x - 4)(x - 1) = 0$$

Either $x = 4$ or $x = 1$

[3 marks]