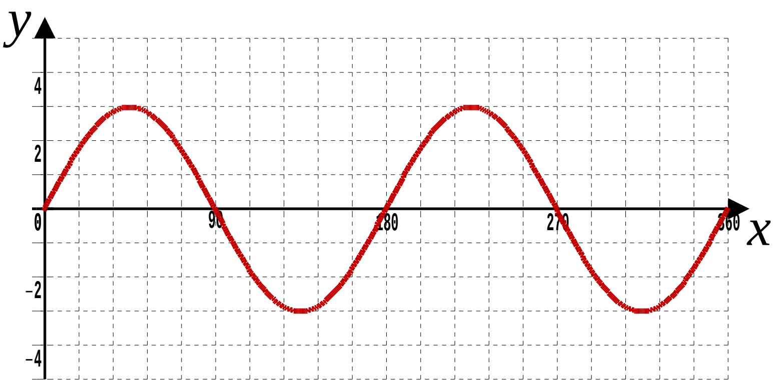
Year 1 Pure Mathematics Examination Revision
Health Check N° 2

Answer 1



[3 marks]

Answer 2
(i)

$$\text{LHS} = \frac{1 - tan^2 x}{1 + tan^2 x}$$

$$= \frac{1 - \frac{sin^2 x}{cos^2 x}}{1 + \frac{sin^2 x}{cos^2 x}}$$

$$= \frac{\left(1 - \frac{sin^2 x}{cos^2 x}\right)}{\left(1 + \frac{sin^2 x}{cos^2 x}\right)} \times \frac{\left(\frac{cos^2 x}{1}\right)}{\left(\frac{cos^2 x}{1}\right)}$$

$$= \frac{cos^2 x - sin^2 x}{cos^2 x + sin^2 x}$$

$$= cos^2 x - sin^2 x$$

$$= cos^2 x - \left(1 - cos^2 x\right)$$

$$= cos^2 x - 1 + cos^2 x$$

$$= 2\, cos^2 x - 1$$

$$= \text{RHS} \qquad \square$$

[6 marks]

(ii)

$$\frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{1}{8}$$

$$2\cos^2 x - 1 = \frac{1}{8} \qquad \text{from part (i)}$$

$$2\cos^2 x = \frac{9}{8}$$

$$\cos^2 x = \frac{9}{16}$$

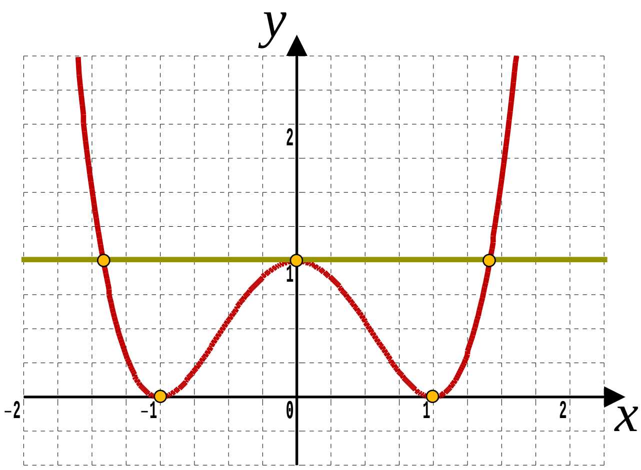
$$\cos x = \pm\frac{3}{4}$$

$$x = 41.4°, \; 138.6°, \; 221.4°, \; 318.6°$$

[5 marks]

Answer 3

(i)



The graph touches the x-axis when $x = \pm 1$ and crosses y-axis when $y = 1$

[5 marks]

(ii)

$$(x - 1)^2 (x + 1)^2 = 1$$

$$((x - 1)(x + 1))^2 = 1$$

$$(x - 1)(x + 1) = \pm 1$$

$$x^2 - 1 = \pm 1$$

$$x^2 = 1 \pm 1$$

$$x^2 = 0 \text{ or } 2$$

$$x = \pm\sqrt{2} \quad \text{or} \quad x = 0$$

[4 marks]

(iii) From part (ii) deduce, $\dfrac{1}{\cos x} = \pm\sqrt{2} \Leftrightarrow \cos x = \pm\dfrac{1}{\sqrt{2}}$

$$\therefore \; x = 45°, \; 135°, \; 225°, \; 315°$$

[4 marks]

Answer 4

(i) $(x - 5)^2 + (y - 12)^2 = 13^2$

This is recognisable as a circle centre (5, 12) and with radius 13.

This is suggesting an alternative way of thinking about circles as being the unit circle centred at the origin being enlarged with scale factor r to the desired radius, and then translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ to be of the form,

$$(x - a)^2 + (y - b)^2 = r^2$$

[3 marks]

(ii) $y = x^2$ after the translation $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ will become,

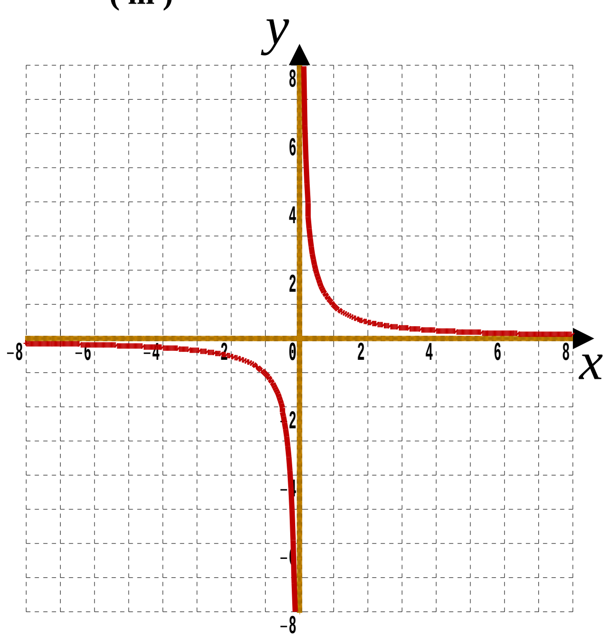
$$(y + 7) = (x - 4)^2$$

$$y = x^2 - 8x + 16 - 7$$

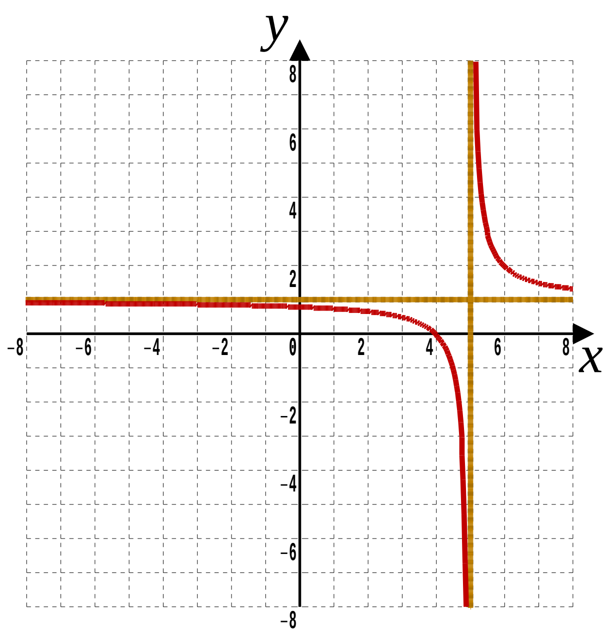
$$y = x^2 - 8x + 9$$

[4 marks]

(iii)



$y = \dfrac{1}{x}$ (red) with asymptotes $x = 0,\ y = 0$



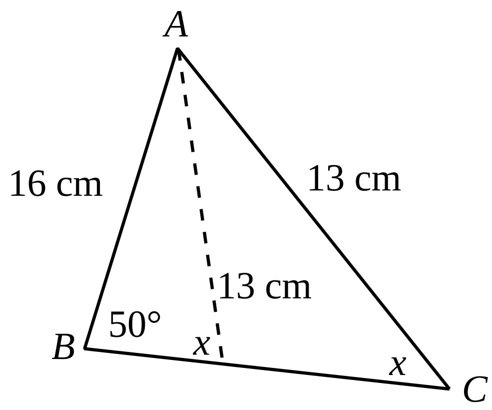
Transformed asymptotes at $x = 5,\ y = 1$

$$y - 1 = \frac{1}{x - 5} \Leftrightarrow y = \frac{1}{x - 5} + 1$$

$$y = \frac{x - 4}{x - 5} \quad \text{so } a = c = 1,\ b = -4,\ d = -5$$

[5 marks]

Answer 5



By the sine rule...

$$\frac{\sin x}{16} = \frac{\sin 50}{13}$$

$$\sin x = \frac{16 \sin 50}{13}$$

$$= 0.9428...$$

$$\therefore\ x = \arcsin 0.9428...$$

$$= 70.5° \quad \text{or} \quad 109.5°$$

[4 marks]

Answer 6

(i) As the parabola cuts the *x*-axis at $(-5,\ 0)$ and $(4,\ 0)$,

$$y = k(x + 5)(x - 4)$$

Use the *y*-axis crossing point at $(0, -5)$ to get the scaling factor, *k*,

$$-5 = k(5)(-4)$$

$$k = \frac{1}{4}$$

$\therefore$ The parabola's equation is $y = \frac{1}{4}(x + 5)(x - 4)$

[4 marks]

(ii) Replacing x with $x - 3$ will translate the graph $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and the solutions to the equation will be where this translated graph cuts the *x*-axis.

$$\therefore\ x = -2 \text{ or } x = 7$$

[3 marks]