Year 1 Pure Mathematics Examination Revision
Health Check N° 1

Statistically, nine out of ten injections are in vein.

Questions kindly provided by Jeremy Lucas

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 70

Question 1

(i) Complete the square for $x^2 - 8x - 13$

[2 marks]

(ii) Hence state the minimum value of the function,

$$f(x) = x^2 - 8x - 13, \qquad x \in \mathbb{R}$$

[1 mark]

(iii) Also state the value of x for which the minimum value occurs.

[1 mark]

Question 2

(i) Expand $(1 - 2x)^8$ in ascending powers of x up to and including the term in x^3. Simplify each term where it's appropriate to do so.

[3 marks]

(ii) Hence find an approximation to the value of 0.98^8 to four decimal places.

[2 marks]

Question 3

(i) Solve the equation $\sin 2x = 0.5$ for $0 \leqslant x \leqslant 360°$

[4 marks]

(ii) Solve the equation $\sin^2 x = \cos x - 1$ for $-360 \leqslant x \leqslant 360$

[4 marks]

Question 4

(i) Differentiate $y = \dfrac{x^2 + 2x + 3}{x}$

[4 marks]

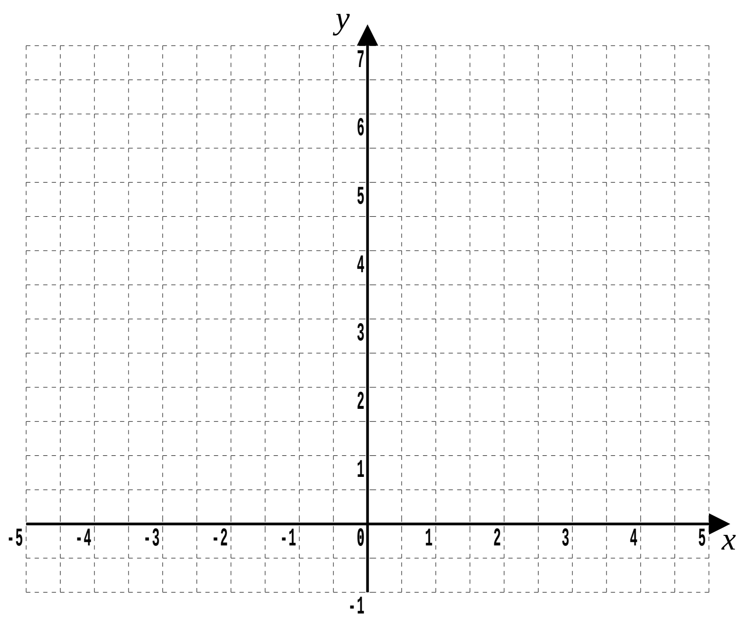
(ii) Integrate $\displaystyle\int \frac{4x - 9}{\sqrt{x}}\, dx$

[4 marks]

Question 5

On the graph paper sketch **(i)** $y = (x - 2)^2$

(ii) $y = (x + 4)(x + 1)^2$



[5 marks]

Question 6

A triangle has sides of length 4 cm, 5 cm and 6 cm.

(i) Find the size of the smallest angle in the triangle.
Give your answer correct to 3 significant figures.

[3 marks]

(ii) Find the area of the triangle.
Give your answer correct to 3 significant figures.

[3 marks]

Question 7

Solve the simultaneous equations;

$$x^2 + y^2 = 100$$
$$x - y = 2$$

[4 marks]

Question 8

Solve the inequalities

(i) $4x^2 + 3 < 39$

[3 marks]

(ii) $x^2 + 5x > 14$

[3 marks]

Question 9

A circle has equation $x^2 + y^2 - 6x + 4y - 3 = 0$

(i) Find the centre and radius of the circle

[3 marks]

(ii) Find the equation of the tangent to the circle at (3, 2)

[4 marks]

Question 10

A straight line L_1 is given by the equation $2x + 3y = 13$

(i) Find the equation of the line L_2 which is parallel to L_1 and passes through the point $(1, 5)$

[2 marks]

(ii) Find the equation of the line L_3 which is perpendicular to L_1 and passes through the point $(1, 5)$

[2 marks]

Question 11

A curve is given by the equation $y = x^3 - 3x^2 - 9x + 15$

(i) Find the coordinates of the two turning points

[4 marks]

(ii) Classify each of these turning points as a minimum or a maximum

[2 marks]

Question 12

Solve the following equations,

(i) $2^{2x} - 3 \times 2^{x} + 2 = 0$

[3 marks]

(ii) $2\,log_3(\,x + 2\,) - log_3 x = 2$

[3 marks]

Year 1 Pure Mathematics Examination Revision
Health Check N° 1

Answer 1

(i)

$$x^2 - 8x - 13 = \left[x^2 - 8x \right] - 13$$
$$= \left[(x - 4)^2 - 16 \right] - 13$$
$$= (x - 4)^2 - 29$$

[2 marks]

(ii) The minimum value of the function is – 29

[1 mark]

(iii) The minimum value occurs when $x = 4$

[1 mark]

Answer 2

(i)

$$(1 - 2x)^8 = 1 \times (1)^8 \times (-2x)^0$$
$$+ \; 8 \times (1)^7 \times (-2x)^1$$
$$+ \; 28 \times (1)^6 \times (-2x)^2$$
$$+ \; 56 \times (1)^5 \times (-2x)^3$$
$$+ \ldots$$
$$= 1 - 16x + 112x^2 - 448x^3 \quad \left(\text{Up to the term in } x^3\right)$$

[3 marks]

(ii) We require $1 - 2x = 0.98$

$$2x = 0.02$$
$$x = 0.01$$
$$\therefore \; 0.98^8 \approx 1 - 16 \times 0.01 + 112 \times 0.01^2 - 448 \times 0.01^3$$
$$\approx 1 - 0.16 + 0.0112 - 0.000448$$
$$\approx 0.850752$$
$$\approx 0.8508 \text{ (4 decimal places requested)}$$

[2 marks]

Answer 3

(i)

$$\sin 2x = 0.5$$
$$2x = 30,\ 150,\ 390,\ 510,\ \ldots$$
$$x = 15,\ 75,\ 195,\ 255$$

[4 marks]

(ii) Using the identity $\cos^2 x + \sin^2 x = 1$,

$$\sin^2 x = \cos x - 1$$
$$1 - \cos^2 x = \cos x - 1$$
$$\cos^2 x + \cos x - 2 = 0$$
$$(\cos x + 2)(\cos x - 1) = 0$$

Either $\cos x = -2$ which has no solutions

Or $\cos x = 1$ which has solutions

$x = 0, \pm 360$ over the domain specified

[4 marks]

Answer 4

(i)

$$y = \frac{x^2 + 2x + 3}{x}$$
$$= \frac{x^2}{x} + \frac{2x}{x} + \frac{3}{x}$$
$$= x + 2 + 3x^{-1}$$
$$\frac{dy}{dx} = 1 - 3x^{-2}$$
$$= 1 - \frac{3}{x^2}$$

[4 marks]

(ii)

$$\int \frac{4x - 9}{\sqrt{x}}\, dx = \int \frac{4x}{\sqrt{x}} - \frac{9}{\sqrt{x}}\, dx$$
$$= \int 4x^{\frac{1}{2}} - 9x^{-\frac{1}{2}}\, dx$$
$$= \left[\frac{4 \times 2x^{\frac{3}{2}}}{3} - \frac{9 \times 2x^{\frac{1}{2}}}{1} \right] + c$$
$$= \frac{8}{3}x^{\frac{3}{2}} - 18x^{\frac{1}{2}} + c$$
$$= \frac{2}{3}\sqrt{x}\,(4x - 27) + c$$

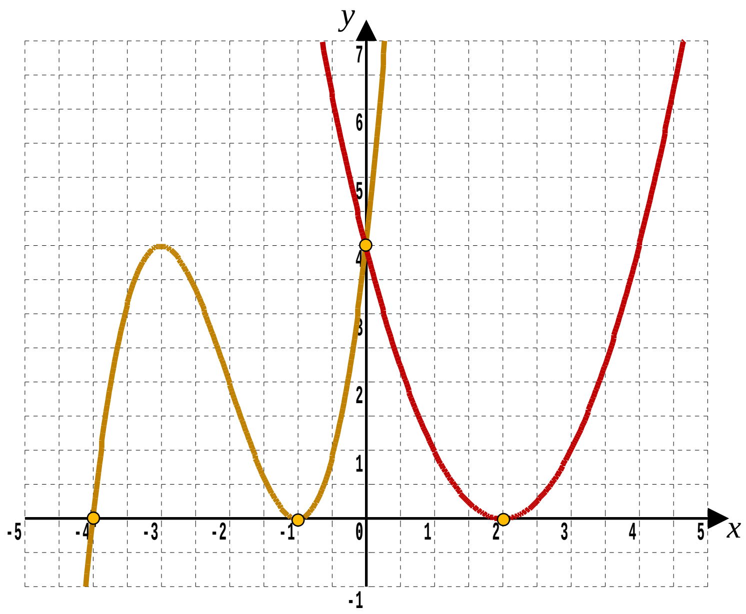
[4 marks]

Answer 5

(i) $y = (x - 2)^2$ is plotted in red

(ii) $y = (x + 4)(x + 1)^2$ is plotted in gold

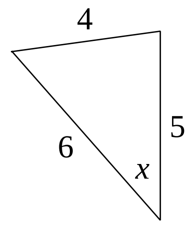
The examiner is looking for the correct overall shape and points where the curves cross the coordinate axes.



[5 marks]

Answer 6

(i) The smallest angle is opposite the shortest side.



By The Cosine Rule: $\cos x = \dfrac{6^2 + 5^2 - 4^2}{2 \times 6 \times 5}$

$$= \frac{3}{4}$$

$$\therefore x = \arccos\left(\frac{3}{4}\right)$$

$$= 41.4°$$

[3 marks]

(ii) $$Area\ \Delta = \frac{1}{2}\, 6 \times 5 \times \sin 41.4°$$

$$= 9.92 \text{ cm}^2$$

[3 marks]

Answer 7

$$x - y = 2 \Rightarrow y = x - 2$$

$$x^2 + y^2 = 100$$

$$x^2 + (x - 2)^2 = 100$$

$$x^2 + x^2 - 4x + 4 = 100$$

$$2x^2 - 4x - 96 = 0$$

divide through by 2

$$x^2 - 2x - 48 = 0$$

$$(x - 8)(x + 6) = 0$$

Either $x = 8$ in which case $y = 6$

Or $x = -6$ in which case $y = -8$

That is, the solutions are $(8, 6)$ or $(-6, -8)$

[4 marks]

Answer 8

(i)

$$4x^2 + 3 < 39$$

$$4x^2 < 36$$

$$x^2 < 9$$

$$-3 < x < 3$$

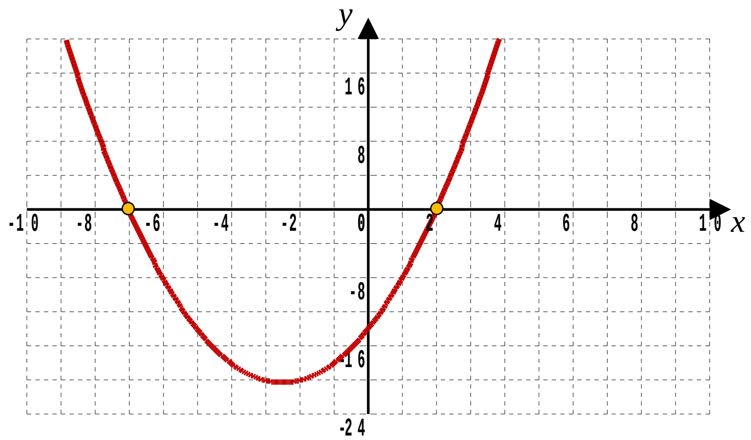
[3 marks]

(ii)

$$x^2 + 5x > 14$$

$$x^2 + 5x - 14 > 0$$

$$(x + 7)(x - 2) > 0$$



Solutions are $x < -7$ or $x > 2$

[3 marks]

Answer 9

(i)

$$x^2 + y^2 - 6x + 4y - 3 = 0$$

$$\left[x^2 - 6x \right] + \left[y^2 + 4y \right] - 3 = 0$$

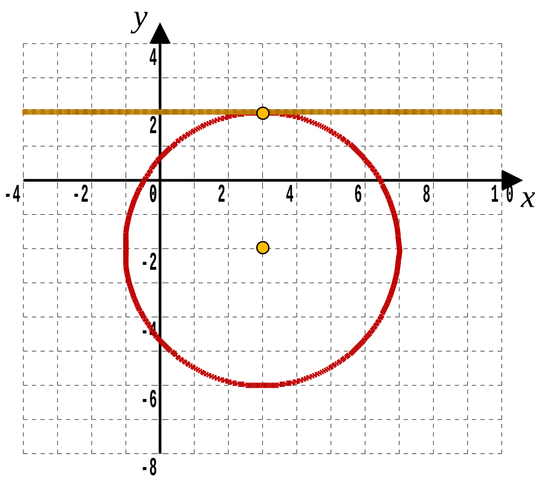
$$\left[(x - 3)^2 - 9 \right] + \left[(y + 2)^2 - 4 \right] - 3 = 0$$

$$(x - 3)^2 + (y + 2)^2 = 16$$

which is a circle with centre $(3, -2)$ and radius 4

[3 marks]

(ii) If you visualise what's going on the answer becomes obvious !



The tangent is $y = 2$

[4 marks]

Answer 10

(i)

$$2x + 3y = 13$$

$$3y = -2x + 13$$

$$y = -\frac{2}{3}x + \frac{13}{3}$$

$\therefore$ L_2 has gradient $-\dfrac{2}{3}$ and passes through $(1, 5)$

$$y = mx + c$$

$$5 = -\frac{2}{3} \times 1 + c$$

$$c = \frac{17}{3}$$

$$y = -\frac{2}{3}x + \frac{17}{3} \quad \text{or} \quad 3y + 2x = 17$$

[2 marks]

(ii) L_3 has gradient $\frac{3}{2}$ and passes through $(1,\ 5)$

$$y = mx + c$$

$$5 = \frac{3}{2} \times 1 + c$$

$$c = \frac{7}{2}$$

$$y = \frac{3}{2}x + \frac{7}{2} \quad \text{or} \quad 2y - 3x = 7$$

[2 marks]

Answer 11

(i)

$$y = x^3 - 3x^2 - 9x + 15$$

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

At turning points $\frac{dy}{dx} = 0$

$$\therefore\ 3x^2 - 6x - 9 = 0$$

divide through by 3

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

Either $x = 3$ in which case $y = -12$

Or $x = -1$ in which case $y = 20$

That is, the turning points are $(-1,\ 20)$, $(3, -12)$

[4 marks]

(ii) From a knowledge of the shape of a cubic curve

$(-1,\ 20)$ is a maximum and $(3, -12)$ is a minimum

Alternatively,

$$\frac{d^2y}{dx^2} = 6x - 6$$

When $x = -1$, $\frac{d^2y}{dx^2} = -12$, i.e. negative $\Rightarrow$ $(-1,\ 20)$ is a maximum

When $x = 3$, $\frac{d^2y}{dx^2} = 12$, i.e. positive $\Rightarrow$ $(3, -12)$ is a minimum

[2 marks]

Answer 12

(i)

$$2^{2x} - 3 \times 2^{x} + 2 = 0$$

$$\left(2^{x} \right)^{2} - 3\left(2^{x} \right) + 2 = 0 \qquad \text{“Quadratic in disguise”}$$

$$\left(2^{x} - 2 \right)\left(2^{x} - 1 \right) = 0$$

Either $2^{x} = 2$ in which case $x = 1$

Or $2^{x} = 1$ in which case $x = 0$

[3 marks]

(ii)

$$2\, log_{3}(x + 2) - log_{3} x = 2$$

$$log_{3}(x + 2)^{2} - log_{3} x = 2$$

$$log_{3}\left(\frac{(x + 2)^{2}}{x} \right) = 2$$

$$\therefore \quad \frac{(x + 2)^{2}}{x} = 3^{2}$$

$$x^{2} + 4x + 4 = 9x$$

$$x^{2} - 5x + 4 = 0$$

$$(x - 4)(x - 1) = 0$$

Either $x = 4$ or $x = 1$

[3 marks]

Year 1 **Pure Mathematics Examination Revision**
Health Check N° 2

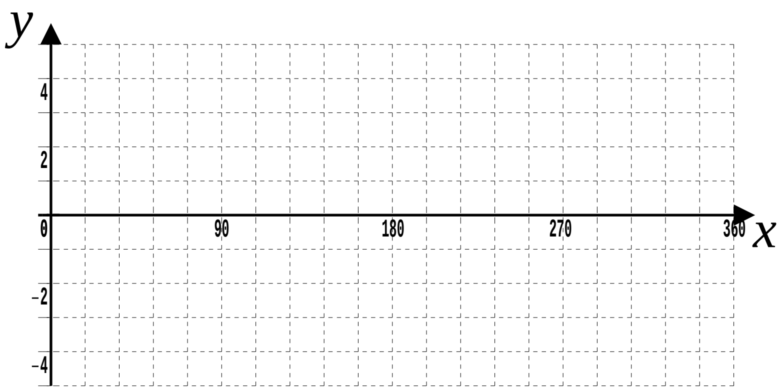
I went to the Library to get a medical book on abdominal pain.
Somebody had ripped the appendix out...

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 50

Question 1

Given a curve, $y = f(x)$, if y is replaced with $\frac{y}{3}$ all distances from the x-axis are tripled and if x is replaced with $2x$ all distances from the y-axis halve.

Use these facts to sketch the graph of $\frac{y}{3} = sin(2x)$ on the grid below.



[3 marks]

Question 2

(i) Show that $\dfrac{1 - tan^2 x}{1 + tan^2 x} = 2\,cos^2 x - 1$

[6 marks]

(ii) Hence solve $\dfrac{1 - tan^2 x}{1 + tan^2 x} = \dfrac{1}{8}$ over the interval $0° \leqslant x \leqslant 360°$

[5 marks]

Question 3

(i) On the same graph sketch the curve $y = (x - 1)^2 (x + 1)^2$ and the straight line $y = 1$, paying particular attention to any points where an axis is touched or crossed.

[5 marks]

(ii) Find all solutions to the equation, $(x - 1)^2 (x + 1)^2 = 1$

[4 marks]

(iii) Solve $\left(\dfrac{1}{\cos x} - 1 \right)^2 \left(\dfrac{1}{\cos x} + 1 \right)^2 = 1, \quad 0 \leqslant x \leqslant 360, \; x \neq 90, \; 270$

[4 marks]

Question 4

To translate any curve by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$

- replace x with $x - a$
- replace y with $y - b$

(i) Given that the equation of a circle centre (0, 0), radius r is $x^2 + y^2 = r^2$

deduce the equation of a circle of radius 13 that has been translated $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$

[3 marks]

(ii) The parabola $y = x^2$ is to be translated $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$

What is the equation of the translated parabola ?

Give your answer in the form $y = a x^2 + bx + c$ where a, b and c are integers the values of which you have determined.

[4 marks]

(iii) The inverse proportion graph, $y = \dfrac{1}{x}$ is translated so that the asymptotes are at $x = 5$ and $y = 1$.

Find the equation of the transformed graph in the form $y = \dfrac{ax + b}{cx + d}$

where *a*, *b*, *c* and *d* are integers, the values of which you have determined.

[5 marks]

Question 5

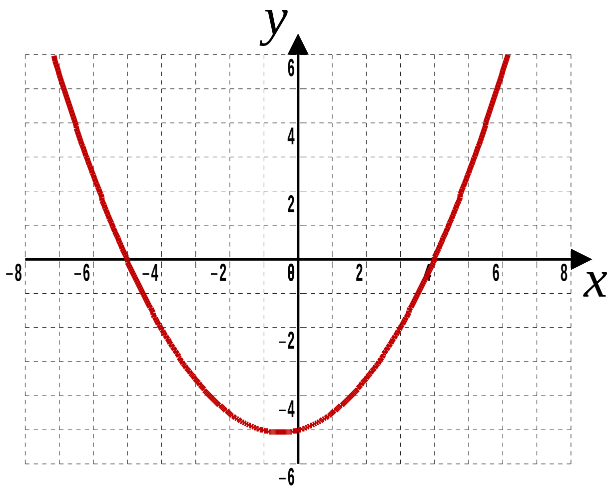
In $\triangle ABC$, AB = 16 cm, AC = 13 cm, $\angle ABC$ = 50° and $\angle BCA = x°$

Find the two possible values for *x*, giving your answers to one decimal place.

[4 marks]

Question 6

The parabola shown below crosses the x-axis at $(-5,\ 0)$ and $(4,\ 0)$ and it crosses the y-axis at $(0, -5)$



(i) Determine the equation of the parabola in the form $y = f(x)$

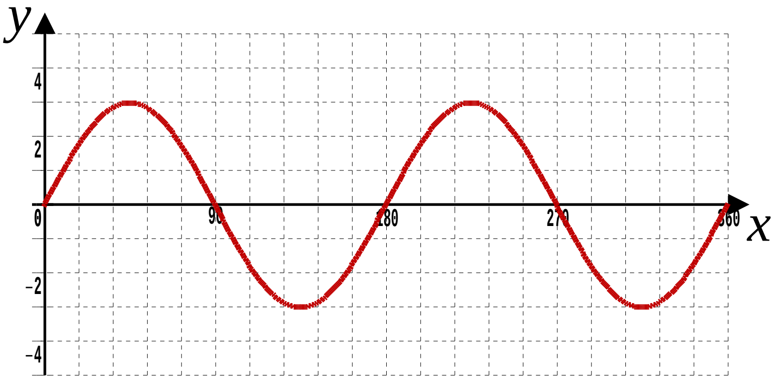
[4 marks]

(ii) Solve the related equation $f(x - 3) = 0$

[3 marks]

Year 1 Pure Mathematics Examination Revision
Health Check N° 2

Answer 1



[3 marks]

Answer 2
(i)

$$\text{LHS} = \frac{1 - tan^2 x}{1 + tan^2 x}$$

$$= \frac{1 - \frac{sin^2 x}{cos^2 x}}{1 + \frac{sin^2 x}{cos^2 x}}$$

$$= \frac{\left(1 - \frac{sin^2 x}{cos^2 x}\right)}{\left(1 + \frac{sin^2 x}{cos^2 x}\right)} \times \frac{\left(\frac{cos^2 x}{1}\right)}{\left(\frac{cos^2 x}{1}\right)}$$

$$= \frac{cos^2 x - sin^2 x}{cos^2 x + sin^2 x}$$

$$= cos^2 x - sin^2 x$$

$$= cos^2 x - \left(1 - cos^2 x\right)$$

$$= cos^2 x - 1 + cos^2 x$$

$$= 2\,cos^2 x - 1$$

$$= \text{RHS} \qquad \square$$

[6 marks]

(ii)

$$\frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{1}{8}$$

$$2\cos^2 x - 1 = \frac{1}{8} \qquad \text{from part (i)}$$

$$2\cos^2 x = \frac{9}{8}$$

$$\cos^2 x = \frac{9}{16}$$

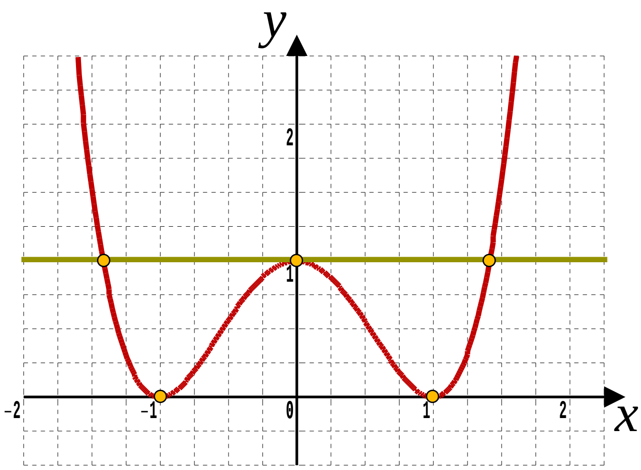
$$\cos x = \pm\frac{3}{4}$$

$$x = 41.4°,\ 138.6°,\ 221.4°,\ 318.6°$$

[5 marks]

Answer 3

(i)



The graph touches the x-axis when $x = \pm 1$ and crosses y-axis when $y = 1$

[5 marks]

(ii)

$$(x - 1)^2 (x + 1)^2 = 1$$

$$((x - 1)(x + 1))^2 = 1$$

$$(x - 1)(x + 1) = \pm 1$$

$$x^2 - 1 = \pm 1$$

$$x^2 = 1 \pm 1$$

$$x^2 = 0 \text{ or } 2$$

$$x = \pm\sqrt{2} \quad \text{or} \quad x = 0$$

[4 marks]

(iii) From part (ii) deduce, $\frac{1}{\cos x} = \pm\sqrt{2} \Leftrightarrow \cos x = \pm\frac{1}{\sqrt{2}}$

$$\therefore \ x = 45°,\ 135°,\ 225°,\ 315°$$

[4 marks]

Answer 4

(i) $(x-5)^2+(y-12)^2=13^2$

This is recognisable as a circle centre (5, 12) and with radius 13.

This is suggesting an alternative way of thinking about circles as being the unit circle centred at the origin being enlarged with scale factor *r* to the desired radius, and then translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ to be of the form,

$$(x-a)^2+(y-b)^2=r^2$$

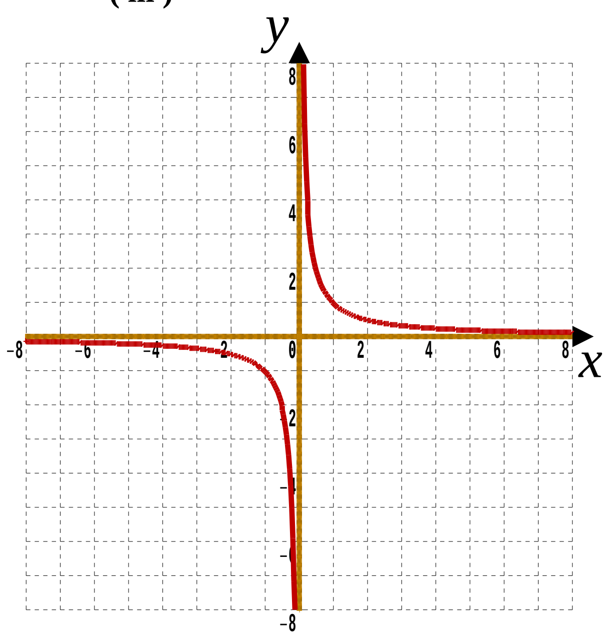
[3 marks]

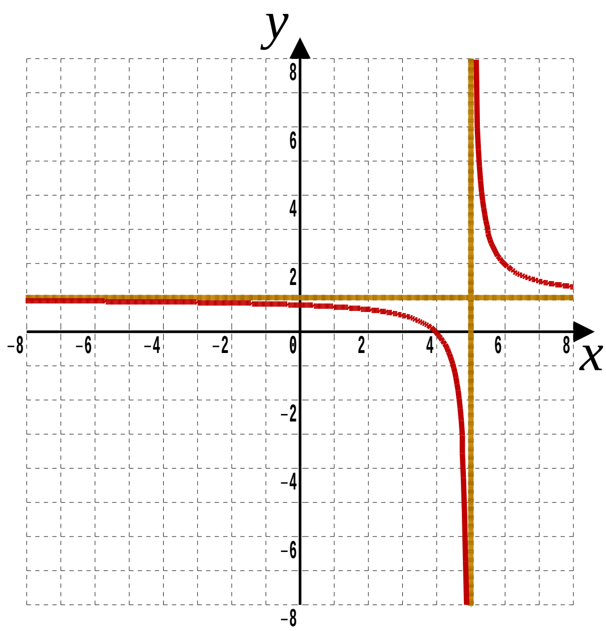
(ii) $y=x^2$ after the translation $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ will become,

$$(y+7)=(x-4)^2$$
$$y=x^2-8x+16-7$$
$$y=x^2-8x+9$$

[4 marks]

(iii)





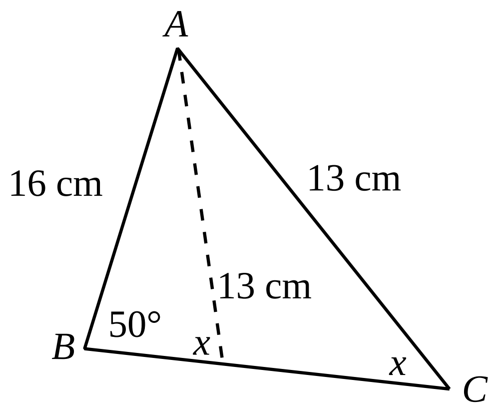
$y=\frac{1}{x}$ (red) with asymptotes $x=0,\ y=0$ Transformed asymptotes at $x=5,\ y=1$

$$y-1=\frac{1}{x-5}\Leftrightarrow y=\frac{1}{x-5}+1$$

$$y=\frac{x-4}{x-5}\quad \text{so } a=c=1,\ b=-4,\ d=-5$$

[5 marks]

Answer 5



By the sine rule...

$$\frac{\sin x}{16} = \frac{\sin 50}{13}$$

$$\sin x = \frac{16 \sin 50}{13}$$

$$= 0.9428...$$

$$\therefore\ x = \arcsin 0.9428...$$

$$= 70.5° \quad \text{or} \quad 109.5°$$

[4 marks]

Answer 6

(i) As the parabola cuts the x-axis at $(-5,\ 0)$ and $(4,\ 0)$,

$$y = k(x + 5)(x - 4)$$

Use the y-axis crossing point at $(0, -5)$ to get the scaling factor, k,

$$-5 = k(5)(-4)$$

$$k = \frac{1}{4}$$

$\therefore$ The parabola's equation is $y = \frac{1}{4}(x + 5)(x - 4)$

[4 marks]

(ii) Replacing x with $x - 3$ will translate the graph $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and the solutions to the equation will be where this translated graph cuts the x-axis.

$$\therefore\ x = -2 \ \text{or}\ x = 7$$

[3 marks]

Year 1 Pure Mathematics Examination Revision
Health Check N° 3

When you get a bladder infection, urine trouble...

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 50

Question 1

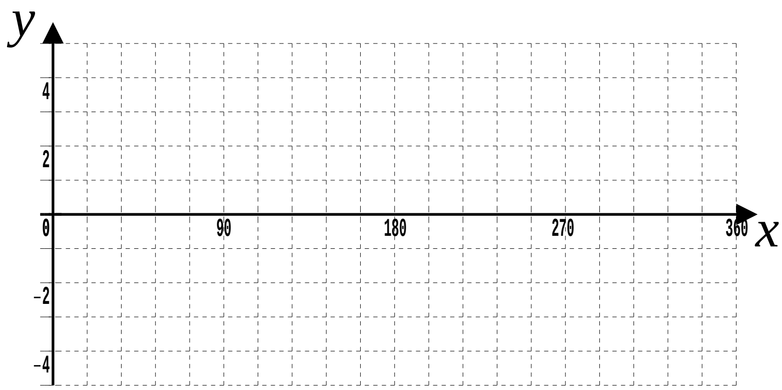
Show that, $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

[4 marks]

Question 2

Given a curve, $y = f(x)$, if y is replaced with $\frac{y}{4}$ all distances from the x-axis are quadrupled and if x is replaced with $(x - 36°)$ the graph translates $\begin{pmatrix} 36 \\ 0 \end{pmatrix}$

Use these facts to sketch the graph of $y = 4\,sin(x - 36)$ on the grid below.



[3 marks]

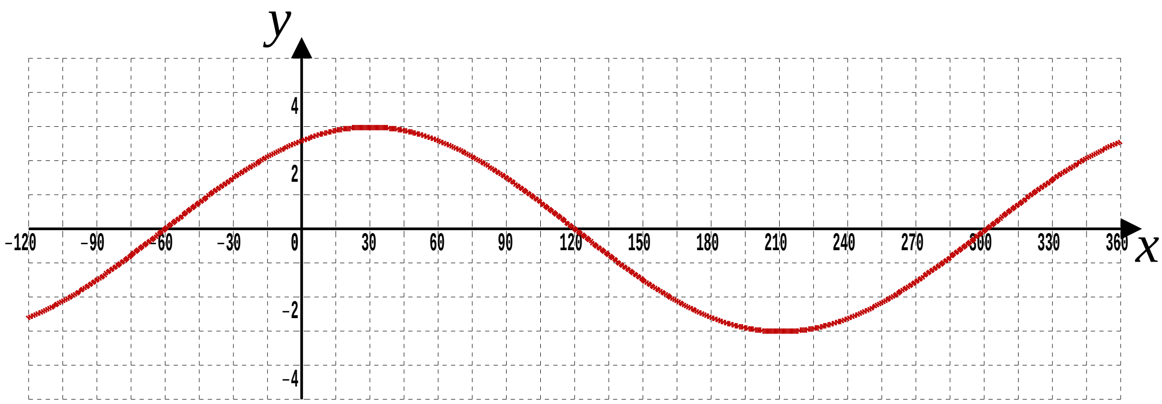
Question 3

Expand the brackets and simplify, $\dfrac{(1 + x)^8 - (1 - x)^8}{16}$

[5 marks]

Question 4

The curve shown has an equation of the form $y = a\sin(x + b)$ for some constants a and b and where x is measured in degrees.



(i) Find the value of a giving a clear reason for your answer.

[2 marks]

(ii) Find the value of b giving a clear reason for your answer.

[2 marks]

(iii) Consider solving an equation of the form,

$$a\sin(x + b) = k,$$

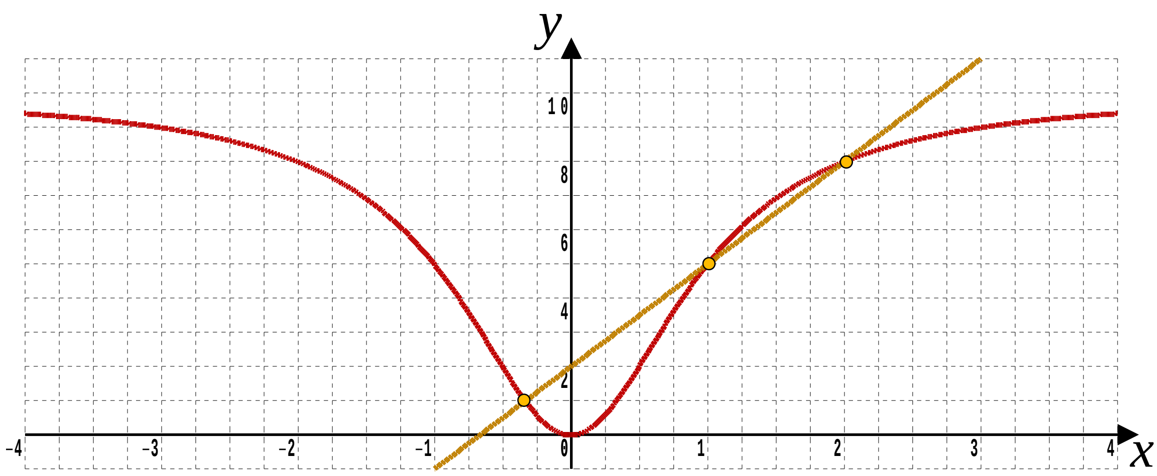
where k is a positive constant.

State the range of possible values for k for which the equation has solutions. Again, give a reason for your answer.

[2 marks]

Question 5

Peyton has graphed the curve $y = \dfrac{10x^2}{x^2 + 1}$ and the straight line $y = 3x + 2$



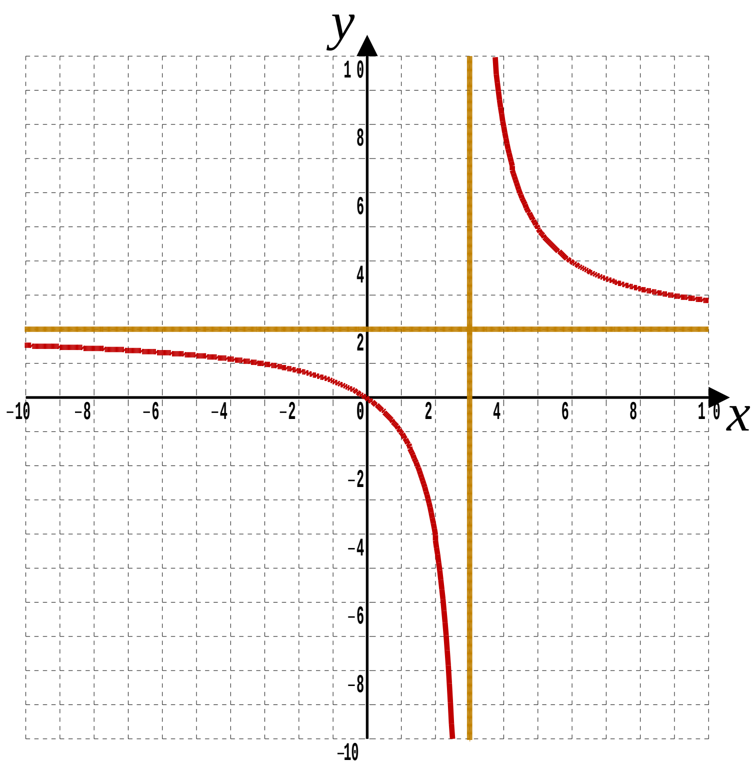
Use algebra to determine the exact coordinates of all three points of intersection.

[6 marks]

Question 6

The graph of inverse proportionality has been translated such that its asymptotes are $x = 3$ and $y = 2$.

Furthemore the translated graph passes through the origin.



Determine the equation of the translated curve in the form $y = \dfrac{ax + b}{cx + d}$ where a, b, c and d are integers, the values of which you have determined.

[6 marks]

Question 7

Oxford University Mathematics Admission Test, 2017, Q(B)

(i) By completing the square, or otherwise, find the minimum value of,

$$f(x) = 9\cos^4 x - 12\cos^2 x + 7$$

[6 marks]

(ii) Find the four values of x between 0° and 360° that give rise to the minimum value of,

$$f(x) = 9\cos^4 x - 12\cos^2 x + 7$$

Give your answers to one decimal place.

[4 marks]

Question 8

From applying the binomial theorem it is known that the first three terms of,

$$\left(1 + \frac{x}{3}\right)^{18} = 1 + 6x + 17x^2 + \ldots$$

Use this expansion to find an estimate of the value of 1.01^{18}, giving your answer to 4 decimal places.

[4 marks]

Question 9

Oxford University Mathematics Admission Test, 2010, Q(A)

Find the values of *k* for which the straingh line with equation $y = kx$ intersects the parabola with equation $y = (x - 1)^2$

[6 marks]

Year 1 Pure Mathematics Examination Revision
Health Check N° 3

Answer 1 (Version 1)

Statement: $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

Proof : LHS $= \cos^4 x - \sin^4 x$

$$= (\cos^2 x)^2 - (\sin^2 x)^2$$

$$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) \quad \text{difference of two squares}$$

$$= \cos^2 x - \sin^2 x$$

$$= (1 - \sin^2 x) - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$= \text{RHS} \qquad \square$$

[4 marks]

Answer 1 (Version 2)

Statement: $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

Proof : LHS $= \cos^4 x - \sin^4 x$

$$= (1 - \sin^2 x)^2 - \sin^4 x$$

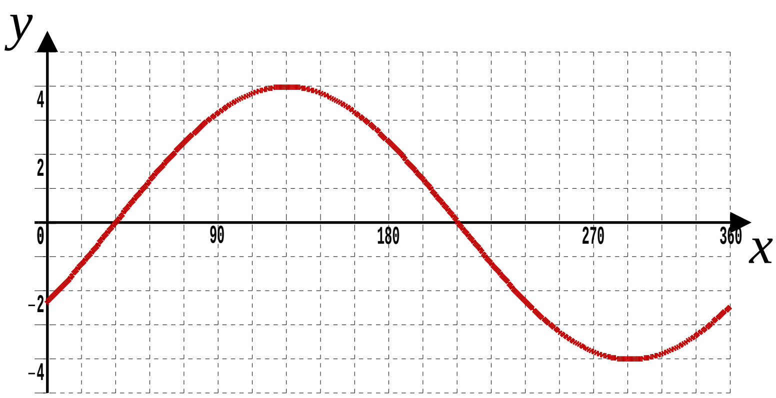
$$= 1 - 2\sin^2 x + \sin^4 x - \sin^4 x$$

$$= 1 - 2\sin^2 x$$

$$= \text{RHS} \qquad \square$$

[4 marks]

Answer 2



[3 marks]

Answer 3

Using the binomial theorem,

$$(1+x)^8 = 1 + 8x + 28x^2 + 56x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + x^8$$

$$(1-x)^8 = 1 - 8x + 28x^2 - 56x^3 + 70x^4 - 56x^5 + 28x^6 - 8x^7 + x^8$$

$$\therefore\ (1+x)^8 - (1-x)^8 = 16x + 112x^3 + 112x^5 + 16x^7$$

$$= 16\left(x + 7x^3 + 7x^5 + x^7\right)$$

$$\frac{(1+x)^8 - (1-x)^8}{16} = x + 7x^3 + 7x^5 + x^7$$

[5 marks]

Answer 4

(i) The graph of $y = \sin x$ has values (outputs) between -1 and $+1$

It is said that the amplitude of the sine function is 1.

The graph plotted has an amplitude of 3, achieved by replacing *y* with $\frac{y}{3}$ and both sides have then been multiplied by 3. $\therefore\ a = 3$

$\therefore\ a = 3$

[2 marks]

(ii) The graph of $y = \sin x$ passes through the origin. That point, which would not be moved by the scaling of part (i), can be considered to now be at $(-60,\ 0)$ after a translation of $\begin{pmatrix} -60 \\ 0 \end{pmatrix}$

Algebraically this is achieved by replacing *x* with $(x + 60)$

$\therefore\ b = 60$

[2 marks]

(iii) $a \sin(x+b) = k \Leftrightarrow \sin(x+b) = \frac{k}{a}$

However, $-1 \leqslant \sin\omega \leqslant 1$ for any real ω whatsoever.

$\therefore$ To have solutions; $-a \leqslant k \leqslant a$

[2 marks]

Answer 5

$$\frac{10x^2}{x^2+1} = 3x+2$$

$$10x^2 = (3x+2)(x^2+1)$$

$$10x^2 = 3x^3+2x^2+3x+2$$

$$3x^3-8x^2+3x+2 = 0$$

It's fairly obvious that $x = 1$ is a root by the factor theorem.

$\therefore\ (x-1)$ is a factor.

$$\begin{array}{r|l} & 3x^2 - 5x - 2 \\ \hline x-1 & 3x^3 - 8x^2 + 3x + 2 \\ & 3x^3 - 3x^2 \\ \hline & \quad -5x^2 + 3x \\ & \quad -5x^2 + 5x \\ \hline & \qquad -2x + 2 \\ & \qquad -2x + 2 \\ \hline & \qquad\qquad 0 \end{array}$$

$0 \leftarrow$ As expected !

$$\therefore\ 3x^3-8x^2+3x+2 = (x-1)(3x^2-5x-2)$$

$$= (x-1)(3x+1)(x-2)$$

$\therefore$ The roots are $x \in \left\{-\frac{1}{3},\ 1,\ 2\right\}$

To get the *y* parts of the coordinates of intersections use the $y = 3x+2$ to get $\left(-\frac{1}{3},\ 1\right)$, $(1,\ 5)$, $(2,\ 8)$

[6 marks]

Answer 6

As *y* is inversely proportional to *x*; $y = \frac{k}{x}$ where *k* is the constant of proportionality.

After a translation of $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ the algebra to go with the graph will be,

$$y-2 = \frac{k}{(x-3)}$$

As it passes through the origin, $-2 = \frac{k}{-3}$ and so $k = 6$

$\therefore\ y-2 = \frac{6}{x-3}$ leading to $y = \frac{2x}{x-3}$

That is, $a = 2,\ b = 0,\ c = 1$ and $d = -3$

[6 marks]

Answer 7

(i)

$$
\begin{aligned}
f(x) &= 9\cos^4 x - 12\cos^2 x + 7 \\
&= \left[\, 9\cos^4 x - 12\cos^2 x \,\right] + 7 \\
&= 9\left[\, \cos^4 x - \frac{4}{3}\cos^2 x \,\right] + 7 \\
&= 9\left[\, \left(\cos^2 x - \frac{2}{3}\right)^2 - \frac{4}{9} \,\right] + 7 \\
&= 9\left(\cos^2 x - \frac{2}{3}\right)^2 - 4 + 7 \\
&= 9\left(\cos^2 x - \frac{2}{3}\right)^2 + 3
\end{aligned}
$$

$\therefore$ The minimum value is 3

[6 marks]

(ii) The minimum value will occur when,

$$
\begin{aligned}
\cos^2 x &= \frac{2}{3} \\
\cos x &= \pm\sqrt{\frac{2}{3}} \\
x &\in \{\, 35.3°,\ 144.7°,\ 215.3°,\ 324.7° \,\}
\end{aligned}
$$

[4 marks]

Answer 8

$$
\begin{aligned}
1 + \frac{x}{3} &= 1.01 \\
\frac{x}{3} &= 0.01 \\
x &= 0.03 \\
\therefore\ 1.01^{18} &\approx 1 + 6 \times 0.03 + 17 \times 0.03^2 \\
1.01^{18} &\approx 1 + 0.18 + 0.0153 \\
1.01^{18} &\approx 1.1953 \qquad \left(\text{Compare with} 1.01^{18} = 1.196147476...\right)
\end{aligned}
$$

[4 marks]

Answer 9

Points of intersection will occur when the following has solutions,

$$(x - 1)^2 = kx$$

$$x^2 - 2x + 1 = kx$$

$$x^2 - (2 + k)x + 1 = 0$$

$$\text{For solutions the discrimiant, } D \geqslant 0$$

$$(-(2 + k))^2 - 4 \times 1 \times 1 \geqslant 0$$

$$4 + 4k + k^2 - 4 \geqslant 0$$

$$k^2 + 4k \geqslant 0$$

$$k(k + 4) \geqslant 0$$

$$\therefore\ k \leqslant -4 \text{ or } k \geqslant 0$$

Geometrically this is interesting and illuminating to try and understand.
Try drawing a picture with the parabola touching the x-axis at $x = 1$ and crossing the y-axis at $y = 1$
Remember that lines in the first quadrant have gradients, m, that satisfy,

$$0 \leqslant m < \infty$$

Can you explain what the maths is telling you about which straight lines through the origin will intersect the parabola ?

[6 marks]