3.5 Answers to 3.4 Exercise

Further A-Level Pure Mathematics : Core 1
Volumes of Revolution

Answer 1

(i)
$$\begin{aligned} Volume &= \pi \int x^2 \, dy \\ &= \pi \int_0^2 \left(6\, e^{3y} \right)^2 dy \\ &= \pi \int_0^2 36\, e^{6y} \, dy \\ &= 6\pi \int_0^2 6\, e^{6y} \, dy \\ &= 6\pi \left[e^{6y} \right]_0^2 \\ &= 6\pi \left(e^{12} - e^0 \right) \\ &= 6\pi \left(e^{12} - 1 \right) \qquad \square \end{aligned}$$

[5 marks]

(ii) 3 067 836.693

[1 mark]

Answer 2

(i)
$$\begin{aligned} Volume &= \pi \int x^2 \, dy \\ &= \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{3}{\cos\left(0.5y\right)} \right)^2 dy \\ &= 9\pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} sec^2 \left(\frac{y}{2} \right) dy \\ &= 18\pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\frac{1}{2} \right) sec^2 \left(\frac{y}{2} \right) dy \\ &= 18\pi \left[tan\left(\frac{y}{2} \right) \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \\ &= 18\pi \left[tan\left(\frac{\pi}{4} \right) - tan\left(\frac{\pi}{6} \right) \right] \\ &= 18\pi \left[1 - \frac{\sqrt{3}}{3} \right] \\ &= 6\pi \left(3 - \sqrt{3} \right) \end{aligned}$$

[5 marks]

(ii) 23.900

[1 mark]

Answer 3

$$Volume = \pi \int x^2 \, dy$$

$$4\pi = \pi \int_1^a \left(\sqrt{y}\right)^2 dy$$

$$4 = \int_1^a y \, dy$$

$$4 = \left[\frac{y^2}{2}\right]_1^a$$

$$4 = \frac{a^2}{2} - \frac{1}{2}$$

$$\frac{a^2}{2} = \frac{9}{2}$$

$$a^2 = 9$$

$$a = 3 \qquad \text{(Ignore } a = -3 \text{ as it's to be an upper limit)}$$

[5 marks]

Answer 4

$$Volume = \pi \int x^2 \, dy$$

$$= \pi \int_1^4 \left(y + \frac{1}{\sqrt{y}}\right)^2 dy$$

$$= \pi \int_1^4 \left(y^2 + 2y^{\frac{1}{2}} + \frac{1}{y}\right) dy$$

$$= \pi \left[\frac{y^3}{3} + \frac{4y^{\frac{3}{2}}}{3} + \ln y\right]_1^4$$

$$= \pi \left\{\left[\frac{64}{3} + \frac{32}{3} + \ln 4\right] - \left[\frac{1}{3} + \frac{4}{3} + \ln 1\right]\right\}$$

$$= \pi \left\{\left[\frac{96}{3} + \ln 4\right] - \left[\frac{5}{3}\right]\right\}$$

$$= \pi \left\{\frac{91}{3} + \ln 4\right\}$$

That is, $K = \frac{91}{3}$

[5 marks]

Answer 5

(i)

$$\frac{dx}{dy} = \ln y + 1$$

[2 marks]

(ii)

$$\int_1^8 (1 + \ln y)\, dy = \left[y \ln y \right]_1^8$$

$$= 8 \ln 8 - \ln 1$$

$$= 8 \ln 2^3 - 0$$

$$= 24 \ln 2 \qquad \square$$

[3 marks]

(iii)

$$24 \pi \ln 2$$

[1 mark]

Answer 6

$$Volume = \pi \int x^2 \, dy$$

$$= \pi \int_0^{4 \ln 3} \left(\frac{1}{4} e^{\frac{y}{2}} \right)^2 dy$$

$$= \frac{\pi}{16} \int_0^{4 \ln 3} e^y \, dy$$

$$= \frac{\pi}{16} \left[e^y \right]_0^{4 \ln 3}$$

$$= \frac{\pi}{16} \left[e^{4 \ln 3} - e^0 \right]$$

$$= \frac{\pi}{16} \left[e^{\ln 3^4} - 1 \right]$$

$$= \frac{\pi}{16} \left[81 - 1 \right]$$

$$= 5 \pi \qquad \square$$

[6 marks]

Answer 7

$$Volume = \pi \int x^2 \, dy$$

$$= \pi \int_0^{\frac{\pi}{6}} \frac{1}{9} \sin\left(\frac{y}{2}\right) dy$$

$$= \frac{\pi}{9} \left[-2 \cos\left(\frac{y}{2}\right) \right]_0^{\frac{\pi}{6}}$$

$$= \frac{\pi}{9} \left\{ \left(-2 \cos\left(\frac{\pi}{12}\right) \right) - \left(-2 \cos 0 \right) \right\}$$

$$= 0.0238 \qquad \text{(3 significant figures asked for)}$$

[6 marks]