5.2 Answers

Answer 1

(i)

$$\int_1^5 \frac{12}{\sqrt{3x+1}}\,dx = 4\int_1^5 3\,(3x+1)^{-\frac{1}{2}}\,dx$$

$$= 4\Big[\,2\,(3x+1)^{\frac{1}{2}}\,\Big]_1^5$$

$$= 8\Big[\,(3x+1)^{\frac{1}{2}}\,\Big]_1^5$$

$$= 8\{\,4 - 2\,\}$$

$$= 16$$

[3 marks]

(ii)

$$Volume = \pi \int y^2\,dx$$

$$= \pi \int_1^5 \left(\frac{12}{\sqrt{3x+1}} \right)^2 dx$$

$$= \pi \int_1^5 \frac{144}{3x+1}\,dx$$

$$= 48\pi \int_1^5 3\,(3x+1)^{-1}\,dx$$

$$= 48\pi\,\Big[\,ln\,|\,3x+1\,|\,\Big]_1^5$$

$$= 48\pi\,\{\,ln\,16 - ln\,4\,\}$$

$$= 48\pi\,\{\,ln\,2^4 - ln\,2^2\,\}$$

$$= 48\pi\,\{\,4\,ln\,2 - 2\,ln\,2\,\}$$

$$= 48\pi\,\{\,2\,ln\,2\,\}$$

$$= 96\,\pi\,ln\,2 \qquad \square$$

[6 marks]

Answer 2

$$
\begin{aligned}
Volume &= \pi \int x^2\,dy \\
&= \pi \int_1^4 \left(1 + \frac{1}{2\sqrt{y}}\right)^2 dy \\
&= \pi \int_1^4 \left(1 + y^{-\frac{1}{2}} + \frac{1}{4y}\right) dy \\
&= \pi \left\{ y + 2y^{\frac{1}{2}} + \frac{1}{4} \ln y \right\}_1^4 \\
&= \pi \left\{ \left[4 + 4 + \frac{1}{4} \ln 4 \right] - \left[1 + 2 + \frac{1}{4} \ln 1 \right] \right\} \\
&= \pi \left\{ \left[8 + \frac{1}{4} \ln 2^2 \right] - [\,3\,] \right\} \\
&= \pi \left\{ 5 + \frac{1}{2} \ln 2 \right\} \qquad \square
\end{aligned}
$$

[9 marks]

Answer 3

(i) When $x = 0$,

$$y\,sin\,y + y = 0$$

$$y\,(\,sin\,y + 1\,) = 0$$

Positive solution sought will come from, $sin\,y + 1 = 0$

$$\therefore\ sin\,y = -1$$

$$y = arcsin\,(-1)$$

$$= \frac{3\pi}{2} \text{ as the first positive solution}$$

That is, coordinates are $\left(0,\ \frac{3\pi}{2}\right)$ ☐

[1 mark]

(ii)

$$Volume = \pi \int x^2\,dy$$

$$= \frac{\pi}{2}\int_0^{\frac{3\pi}{2}} y\,sin\,y + y\ dy$$

L I D I

$$= \frac{\pi}{2}\left[\,y\,(-cos\,y)\,\right]_0^{\frac{3\pi}{2}} - \frac{\pi}{2}\int_0^{\frac{3\pi}{2}}(-cos\,y)\ dy + \frac{\pi}{2}\left[\,\frac{y^2}{2}\,\right]_0^{\frac{3\pi}{2}}$$

$$= -\frac{\pi}{2}\left[\,\frac{3\pi}{2} \times 0 - 0\,\right] + \frac{\pi}{2}\left[\,sin\,y\,\right]_0^{\frac{3\pi}{2}} + \frac{\pi}{4}\left[\,\left(\frac{3\pi}{2}\right)^2 - 0\,\right]$$

$$= \frac{\pi}{2}\left[\,sin\left(\frac{3\pi}{2}\right) - sin\,(\,0\,)\,\right] + \frac{9\,\pi^3}{16}$$

$$= \frac{\pi}{2}\left[-1\,\right] + \frac{9\,\pi^3}{16}$$

$$= \frac{\pi}{2}\left(\frac{9\,\pi^2}{8} - 1\right) \quad \text{or} \quad \frac{9\,\pi^3}{16} - \frac{8\pi}{16} \quad \text{or} \quad \frac{\pi}{16}\left(9\pi^2 - 8\right)$$

[8 marks]

Answer 4

$$Volume = \pi \int y^2 \, dx$$

$$= \pi \int_{\frac{1}{2}}^{2} \left(\frac{2\sqrt{2}}{\sqrt{x}\ (5x - 2)} \right)^2 dx$$

$$= \pi \int_{\frac{1}{2}}^{2} \frac{8}{x(5x - 2)^2} \, dx$$

$$\frac{8}{x(5x-2)^2} = \frac{A}{x} + \frac{B}{5x-2} + \frac{C}{(5x-2)^2}$$

$$\frac{8 \times [\dots\dots]}{[\, x(5x-2)^2 \,]} = \frac{A \times [\dots\dots]}{x} + \frac{B \times [\dots\dots]}{5x-2} + \frac{C \times [\dots\dots]}{(5x-2)^2}$$

$$8 = A(5x-2)^2 + Bx(5x-2) + Cx$$

$$\text{Let } x = 0 \ : \ 8 = 4A \qquad \Rightarrow A = 2$$

$$\text{Let } x = \frac{2}{5} \ : \ 8 = \frac{2C}{5} \qquad \Rightarrow C = 20$$

$$\text{Let } x = 1 \ : \ 8 = 9A + 3B + C \qquad \Rightarrow B = -10$$

$$\frac{8}{x(5x-2)^2} = \frac{2}{x} - \frac{10}{5x-2} + \frac{20}{(5x-2)^2}$$

$$\pi \int_{\frac{1}{2}}^{1} \frac{1}{x(5x-2)^2} \, dx = 2\pi \int_{\frac{1}{2}}^{1} \frac{1}{x} \, dx - 2\pi \int_{\frac{1}{2}}^{1} \frac{5}{(5x-2)} \, dx + 4\pi \int_{\frac{1}{2}}^{1} \frac{5}{(5x-2)^2} \, dx$$

$$= 2\pi \left[\ln|x| \right]_{\frac{1}{2}}^{1} - 2\pi \left[\ln|5x-2| \right]_{\frac{1}{2}}^{1} + 4\pi \left[\frac{(-1)}{5x-2} \right]_{\frac{1}{2}}^{1}$$

$$= 2\pi \left[\ln 1 - \ln\left(\frac{1}{2}\right) \right] - 2\pi \left[\ln 3 - \ln\left(\frac{1}{2}\right) \right] - 4\pi \left[\frac{1}{3} - 2 \right]$$

$$= -2\pi \ln\left(\frac{1}{2}\right) - 2\pi \ln 3 + 2\pi \ln\left(\frac{1}{2}\right) + \frac{20\pi}{3}$$

$$= \frac{20\pi}{3} - 2\pi \ln 3$$

$$(= 14.04)$$

[13 marks]