10.4 Answers

10.4.1 Solution to 10.2 Teaching Video (A Pre-Multiply, Post-Multiply Example)

(i)

$$\begin{aligned}
\mathbf{ABC} &= \mathbf{I} && \\
\mathbf{A}^{-1}\mathbf{ABC} &= \mathbf{A}^{-1}\mathbf{I} && \text{Pre-multiply both sides by } \mathbf{A}^{-1} \\
\mathbf{BC} &= \mathbf{A}^{-1} && \text{As } \mathbf{A}^{-1}\mathbf{A} = \mathbf{I} \\
\mathbf{B}^{-1}\mathbf{BC} &= \mathbf{B}^{-1}\mathbf{A}^{-1} && \text{Pre-multiply both sides by } \mathbf{B}^{-1} \\
\mathbf{C} &= \mathbf{B}^{-1}\mathbf{A}^{-1} && \text{As } \mathbf{B}^{-1}\mathbf{B} = \mathbf{I} \\
\mathbf{CA} &= \mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{A} && \text{Post-multipiy both sides by } \mathbf{A} \\
\mathbf{CA} &= \mathbf{B}^{-1} && \text{As } \mathbf{A}^{-1}\mathbf{A} = \mathbf{I} \\
\therefore\ \mathbf{B}^{-1} &= \mathbf{CA} \quad \square &&
\end{aligned}$$

[4 marks]

(ii)

$$\begin{aligned}
\mathbf{B}^{-1} &= \mathbf{CA} \\
&= \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & -6 \end{pmatrix} \\
&= \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\therefore\ \mathbf{B} &= \begin{pmatrix} -1 & -4 \\ 1 & 3 \end{pmatrix}^{-1} \\
&= \frac{1}{(-1)\times(3) - (-4)\times 1}\begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix} \\
&= \frac{1}{(-3) + 4}\begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix} \\
&= \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix}
\end{aligned}$$

[4 marks]

10.4.2 Solution to 10.3 Exercise

Answer 1

(a)

$$\mathbf{A}^{-1} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}^{-1}$$

$$= \frac{1}{2 \times 3 - (-1) \times 4} \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$$

$$= \frac{1}{10} \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3 & 0.1 \\ -0.4 & 0.2 \end{pmatrix}$$

[2 marks]

(b) Told that,

$$\mathbf{P} = \mathbf{AB}$$

$$\mathbf{A}^{-1}\mathbf{P} = \mathbf{A}^{-1}\mathbf{AB} \qquad \text{Pre-multiply both sides by } \mathbf{A}^{-1}$$

$$\mathbf{A}^{-1}\mathbf{P} = \mathbf{B} \qquad \text{As } \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$\therefore \mathbf{B} = \mathbf{A}^{-1}\mathbf{P}$$

$$= \frac{1}{10} \begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

$$= \frac{1}{10} \begin{pmatrix} 20 & 10 \\ 10 & -40 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 1 \\ 1 & -4 \end{pmatrix}$$

[4 marks]

Answer 2

(a) To show that **A** is non-singular, show that *det* **A** $\neq 0$

$$
\begin{aligned}
det\,\mathbf{A} &= det \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \\
&= 0 \times 3 - 1 \times 2 \\
&= -2 \\
&\neq 0
\end{aligned}
$$

$\therefore$ **A** is non-singular

[2 marks]

(b)

$$
\begin{aligned}
\mathbf{B\,A}^2 &= \mathbf{A} \\
\mathbf{BAA} &= \mathbf{A} \\
\mathbf{BAA\,A}^{-1} &= \mathbf{A\,A}^{-1} && \text{Post-multiply both sides by } \mathbf{A}^{-1} \\
\mathbf{BA} &= \mathbf{I} && \text{As } \mathbf{A\,A}^{-1} = \mathbf{I} \\
\mathbf{BA\,A}^{-1} &= \mathbf{I\,A}^{-1} && \text{Post-multiply both sides by } \mathbf{A}^{-1} \\
\mathbf{B} &= \mathbf{A}^{-1} && \text{As } \mathbf{A\,A}^{-1} = \mathbf{I} \\
&= \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}^{-1} \\
&= \frac{1}{(-2)} \begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix} \\
&= \frac{1}{2} \begin{pmatrix} -3 & 1 \\ 2 & 0 \end{pmatrix} \\
&= \begin{pmatrix} -1.5 & 0.5 \\ 1 & 0 \end{pmatrix}
\end{aligned}
$$

[4 marks]

Answer 3

(i)

$$\begin{aligned}
\mathbf{AXBA} &= \mathbf{I} \\
\mathbf{A}^{-1}\,\mathbf{AXBA} &= \mathbf{A}^{-1}\,\mathbf{I} && \text{Pre-multiply both sides by } \mathbf{A}^{-1} \\
\mathbf{XBA} &= \mathbf{A}^{-1} && \text{As } \mathbf{A}^{-1}\mathbf{A} = \mathbf{I} \\
\mathbf{X}^{-1}\,\mathbf{XBA} &= \mathbf{X}^{-1}\,\mathbf{A}^{-1} && \text{Pre-multiply both sides by } \mathbf{X}^{-1} \\
\mathbf{BA} &= \mathbf{X}^{-1}\,\mathbf{A}^{-1} && \text{As } \mathbf{X}^{-1}\mathbf{X} = \mathbf{I} \\
\mathbf{BAA} &= \mathbf{X}^{-1}\,\mathbf{A}^{-1}\,\mathbf{A} && \text{Post-multiply both sides by } \mathbf{A} \\
\mathbf{BAA} &= \mathbf{X}^{-1} && \text{As } \mathbf{A}^{-1}\mathbf{A} = \mathbf{I} \\
\therefore\ \mathbf{X}^{-1} &= \mathbf{BAA} \quad \square
\end{aligned}$$

[4 marks]

(ii)

$$\begin{aligned}
\mathbf{X}^{-1} &= \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 2 & 1 \\ -3 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 2 & 5 \\ -3 & -7 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
\mathbf{X} &= \begin{pmatrix} 2 & 5 \\ -3 & -7 \end{pmatrix}^{-1} \\
&= \frac{1}{2 \times (-7) - 5 \times (-3)} \begin{pmatrix} -7 & -5 \\ 3 & 2 \end{pmatrix} \\
&= \frac{1}{1} \begin{pmatrix} -7 & -5 \\ 3 & 2 \end{pmatrix} \\
&= \begin{pmatrix} -7 & -5 \\ 3 & 2 \end{pmatrix}
\end{aligned}$$

[4 marks]

Answer 4

(a) **(i)**

$$\mathbf{S}^{-1} = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}^{-1}$$

$$= \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}$$

(ii)

$$\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}^{-1}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$$

(iii)

$$\mathbf{BS} = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 4 \\ -4 & -6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}$$

(iv)

$$(\mathbf{BS})^{-1} = \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}^{-1}$$

$$= \frac{1}{1}\begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}$$

[2 marks each = 8 marks in Total]

(b)

$$\text{LHS} = \mathbf{S}^{-1}\mathbf{B}^{-1}$$

$$= \frac{1}{2}\begin{pmatrix} 3 & -4 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -6 & -4 \\ 4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}$$

$$= (\mathbf{BS})^{-1}$$

$$= \text{RHS} \qquad \square$$

[3 marks]

(c) The opposite of putting on socks then putting on boots is first taking off boots then taking off socks.

[2 marks]

Answer 5

(a)

$$\mathbf{A}^{-1} = \begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix}^{-1}$$

$$= \frac{1}{2p \times 5q - 3p \times 3q} \begin{pmatrix} 5q & -3q \\ -3p & 2q \end{pmatrix}$$

$$= \frac{1}{10pq - 9pq} \begin{pmatrix} 5q & -3q \\ -3p & 2q \end{pmatrix}$$

$$= \frac{1}{pq} \begin{pmatrix} 5q & -3q \\ -3p & 2q \end{pmatrix}$$

[3 marks]

(b)

$$\mathbf{XA} = \mathbf{B}$$

$$\mathbf{XA}\,\mathbf{A}^{-1} = \mathbf{B}\,\mathbf{A}^{-1} \qquad \text{Post-multiple both sides by } \mathbf{A}^{-1}$$

$$\mathbf{X} = \mathbf{BA}^{-1} \qquad \text{As } \mathbf{AA}^{-1} = \mathbf{I}$$

$$= \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix} \frac{1}{pq} \begin{pmatrix} 5q & -3q \\ -3p & 2q \end{pmatrix}$$

$$= \frac{1}{pq} \begin{pmatrix} 2pq & -pq \\ -3pq & 4pq \\ pq & pq \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 1 \end{pmatrix}$$

[4 marks]