

11.2 Answers

Further A-Level Pure Mathematics : Core 1 Matrix Transformations

Answer 1

As matrix **M** is singular,

$$\det \mathbf{M} = 0$$

$$\det \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix} = 0$$

$$x \times (4x - 11) - [(x - 2)(3x - 6)] = 0$$

$$4x^2 - 11x - [3x^2 - 12x + 12] = 0$$

$$4x^2 - 11x - 3x^2 + 12x - 12 = 0$$

$$x^2 + x - 12 = 0$$

$$(x + 4)(x - 3) = 0$$

$$\text{Either } x + 4 = 0 \text{ or } x - 3 = 0$$

$$x = -4 \text{ or } x = 3$$

[4 marks]

Answer 2

$$(a) \quad \mathbf{R}^2 = \begin{pmatrix} a & 2 \\ a & b \end{pmatrix} \begin{pmatrix} a & 2 \\ a & b \end{pmatrix} = \begin{pmatrix} a^2 + 2a & 2a + 2b \\ a^2 + ab & 2a + b^2 \end{pmatrix}$$

[3 marks]

(b) The question is telling us that,

$$\begin{pmatrix} a^2 + 2a & 2a + 2b \\ a^2 + ab & 2a + b^2 \end{pmatrix} = \begin{pmatrix} 15 & 0 \\ 0 & 15 \end{pmatrix}$$

$$\text{Upper right : } 2a + 2b = 0 \Rightarrow a = -b$$

$$\text{Upper left : } a^2 + 2a = 15$$

$$a^2 + 2a - 15 = 0$$

$$(a + 5)(a - 3) = 0 \Rightarrow a = 3 \quad \text{as told } a > 0$$

$$\therefore b = -3$$

[5 marks]

Answer 3**(a)**

$$\begin{aligned}
\mathbf{M} &= (\mathbf{A} + \mathbf{B}) (2\mathbf{A} - \mathbf{B}) \\
&= \left(\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \right) \left(2 \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \right) \\
&= \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 5 & 1 \\ -2 & -1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & -1 \\ -7 & -2 \end{pmatrix}
\end{aligned}$$

[6 marks]**(b)**

$$\mathbf{MC} = \mathbf{A}$$

$$\mathbf{M}^{-1} \mathbf{MC} = \mathbf{M}^{-1} \mathbf{A} \quad \text{Pre-multiply both sides by } \mathbf{M}^{-1}$$

$$\mathbf{C} = \mathbf{M}^{-1} \mathbf{A} \quad \text{As } \mathbf{M}^{-1} \mathbf{M} = \mathbf{I}$$

$$= -\frac{1}{9} \begin{pmatrix} -2 & 1 \\ 7 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$$

$$= -\frac{1}{9} \begin{pmatrix} -5 & -2 \\ 13 & 7 \end{pmatrix}$$

$$= \frac{1}{9} \begin{pmatrix} 5 & 2 \\ -13 & -7 \end{pmatrix}$$

[4 marks]

Answer 4**(i) (a)**

$$\begin{aligned}\mathbf{AB} &= \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix} \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix} \\ &= \begin{pmatrix} -5p + 12 & 4p - 10 \\ -15 + 6p & 12 - 5p \end{pmatrix}\end{aligned}$$

[2 marks]**(b)**

$$\mathbf{AB} + 2\mathbf{A} = k\mathbf{I}$$

$$\begin{aligned}\begin{pmatrix} -5p + 12 & 4p - 10 \\ -15 + 6p & 12 - 5p \end{pmatrix} + 2\begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix} &= k \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} -3p + 12 & 4p - 6 \\ -9 + 6p & 12 - 3p \end{pmatrix} &= \begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\end{aligned}$$

$$\text{Upper Right : } 4p - 6 = 0 \Rightarrow p = \frac{3}{2}$$

$$\text{Lower Left : } 12 - 3p = k \Rightarrow k = \frac{15}{2}$$

[4 marks]**(ii)** The question is telling us that,

$$\det \mathbf{M} = \pm \frac{270}{15}$$

$$\det \begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix} = \pm 18$$

$$2a + 9 = \pm 18$$

$$2a = \pm 18 - 9$$

$$\text{Either } a = 4.5 \text{ or } a = -13.5$$

[5 marks]

Answer 5**(i) (a)**

$$\begin{aligned}\mathbf{AB} &= \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 4 & 5 & 6 \\ 5 & -6 & 11 \\ 13 & 11 & 21 \end{pmatrix}\end{aligned}$$

[3 marks]**(b)**

$$\begin{aligned}\mathbf{BA} &= \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix} \\ &= \begin{pmatrix} 15 & 25 \\ 14 & 4 \end{pmatrix}\end{aligned}$$

$$\therefore \mathbf{AB} \neq \mathbf{BA} \quad \square$$

[1 mark]**(ii)**

$$\begin{aligned}\mathbf{C} &= \begin{pmatrix} 2k & -2 \\ 3 & k \end{pmatrix} \\ \mathbf{C}^{-1} &= \frac{1}{2k^2 + 6} \begin{pmatrix} k & 2 \\ -3 & 2k \end{pmatrix}\end{aligned}$$

[3 marks]