

1.4 Answers

Further A-Level Pure Mathematics : Core 1 Matrix Transformations

Answer 1

$$\begin{aligned} \text{(i)} \quad \mathbf{A} + \mathbf{B} &= \begin{pmatrix} 4 & -8 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 9 & 6 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 13 & -2 \\ 2 & -3 \end{pmatrix} \end{aligned}$$

[2 marks]

$$\begin{aligned} \text{(ii)} \quad \mathbf{A} - \mathbf{B} &= \begin{pmatrix} 4 & -8 \\ 2 & 0 \end{pmatrix} - \begin{pmatrix} 9 & 6 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 4 & -8 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} -9 & -6 \\ 0 & 3 \end{pmatrix} \\ &= \begin{pmatrix} -5 & -14 \\ 2 & 3 \end{pmatrix} \end{aligned}$$

[2 marks]

$$\begin{aligned} \text{(iii)} \quad 4\mathbf{A} - 3\mathbf{B} &= 4 \begin{pmatrix} 4 & -8 \\ 2 & 0 \end{pmatrix} - 3 \begin{pmatrix} 9 & 6 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 16 & -32 \\ 8 & 0 \end{pmatrix} + \begin{pmatrix} -27 & -18 \\ 0 & 9 \end{pmatrix} \\ &= \begin{pmatrix} -11 & -50 \\ 8 & 9 \end{pmatrix} \end{aligned}$$

[2 marks]

$$\begin{aligned} \text{(iv)} \quad \frac{1}{2}\mathbf{A} + \frac{2}{3}\mathbf{B} &= \frac{1}{2} \begin{pmatrix} 4 & -8 \\ 2 & 0 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 9 & 6 \\ 0 & -3 \end{pmatrix} \\ &= \begin{pmatrix} 2 & -4 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 6 & 4 \\ 0 & -2 \end{pmatrix} \\ &= \begin{pmatrix} 8 & 0 \\ 1 & -2 \end{pmatrix} \end{aligned}$$

[2 marks]

Answer 2

$$\begin{aligned} \begin{pmatrix} a & 2 \\ -1 & b \end{pmatrix} - \begin{pmatrix} 2 & c \\ d & -2 \end{pmatrix} &= 5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} (a-2) & (2-c) \\ (-1-d) & (b+2) \end{pmatrix} &= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \\ \therefore a = 7, \quad b = 3, \quad c = 2 \text{ and } d = -1 \end{aligned}$$

[4 marks]

Answer 3

$$\begin{pmatrix} 4 & b \\ a & c \end{pmatrix} - \begin{pmatrix} a & 6 \\ a & d \end{pmatrix} = \begin{pmatrix} 11 & a \\ c & b \end{pmatrix}$$
$$\begin{pmatrix} (4 - a) & (b - 6) \\ 0 & (c - d) \end{pmatrix} = \begin{pmatrix} 11 & a \\ c & b \end{pmatrix}$$

$\therefore a = -7, \quad b = -1, \quad c = 0 \text{ and } d = 1$

[4 marks]

Answer 4

$$2k^2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} - 3 \begin{pmatrix} 1 \\ 4k \end{pmatrix} = \begin{pmatrix} k \\ -9 \end{pmatrix}$$
$$\begin{pmatrix} 2k^2 - 3 \\ 4k^2 - 12k \end{pmatrix} = \begin{pmatrix} k \\ -9 \end{pmatrix}$$

From the upper row, $2k^2 - k - 3 = 0$

$$(2k - 3)(k + 1) = 0$$

Either $k = \frac{3}{2}$ or $k = -1$

From the lower row, $4k^2 - 12k + 9 = 0$

$$(2k - 3)(2k - 3) = 0$$

$$k = \frac{3}{2}$$

The only common solution for the system of equations is $k = \frac{3}{2}$

[4 marks]

Answer 5

$$\begin{pmatrix} p & 0 & 0 \\ 0 & q^2 & r \\ 0 & 0 & 5 \end{pmatrix} - k \begin{pmatrix} 2q & 0 & 0 \\ 0 & 4 & 6 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Lower right : $5 - 2k = 1 \quad \Rightarrow k = 2$

Centre right : $r - 6k = 0 \quad \Rightarrow r = 12$

Centre : $q^2 - 4k = 1 \quad \Rightarrow q = 3$

Upper left : $p - 2qk = 1 \quad \Rightarrow p = 13$

[4 marks]

Answer 6

$$\begin{aligned} |\mathbf{A}| &= \left| \begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix} \right| \\ &= 7 \times 3 - 4 \times 5 \\ &= 1 \end{aligned}$$

[1 mark]

Answer 7

$$\begin{aligned} |\mathbf{A}| &= 24 \\ \left| \begin{pmatrix} -2 & k \\ 5 & 3 \end{pmatrix} \right| &= 24 \\ -2 \times 3 - k \times 5 &= 24 \\ -5k &= 30 \\ k &= -6 \end{aligned}$$

[2 marks]

Answer 8

$$\begin{aligned} |\mathbf{A}| &= 0 \\ \left| \begin{pmatrix} 3 & (k+3) \\ (k-2) & 8 \end{pmatrix} \right| &= 0 \\ 3 \times 8 - (k+3)(k-2) &= 0 \\ 24 - (k^2 + k - 6) &= 0 \\ 24 - k^2 - k + 6 &= 0 \\ k^2 + k - 30 &= 0 \\ (k+6)(k-5) &= 0 \\ \text{Either } k &= -6 \text{ or } k = 5 \end{aligned}$$

[3 marks]