3.3 Answers

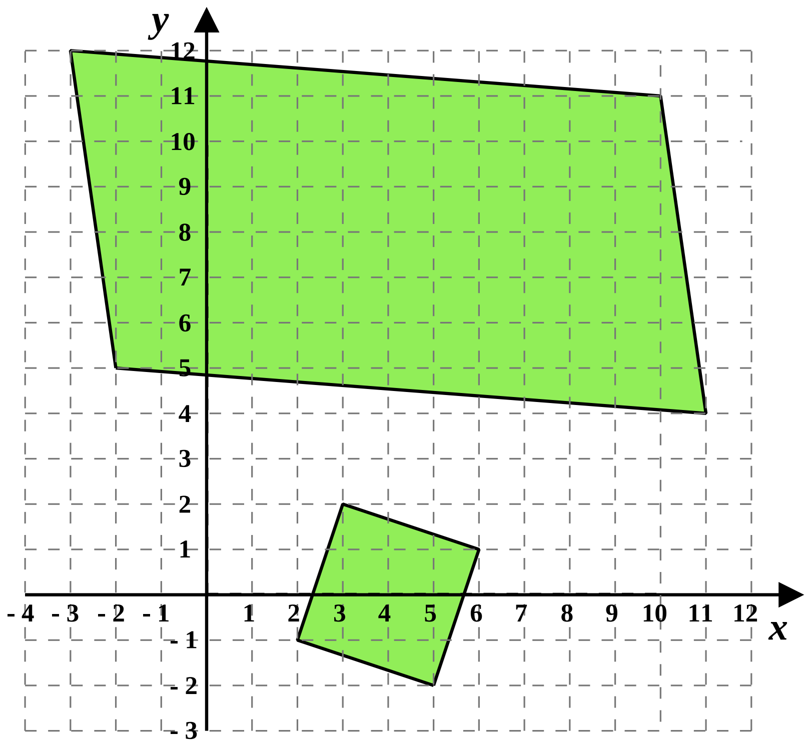
3.3.1 Solution to 3.1 Teaching Video Example

(i) and **(ii)**

$$\begin{array}{c|c} & \begin{pmatrix}3 & 6 & 5 & 2\\ 2 & 1 & -2 & -1\end{pmatrix}\\ \hline \begin{pmatrix}1 & 4\\ 2 & -1\end{pmatrix} & \begin{pmatrix}11 & 10 & -3 & -2\\ 4 & 11 & 12 & 5\end{pmatrix}\end{array}$$

[1, 4 marks]

(iii)



[1 mark]

Note that there is an assumption made that the straight lines between the vertices of the original shape transform to straight lines between the vertices of the transformed shape.
This is indeed what happens - a challenge question would be to prove this !

3.3.2 Solutions to 3.2 Exercise

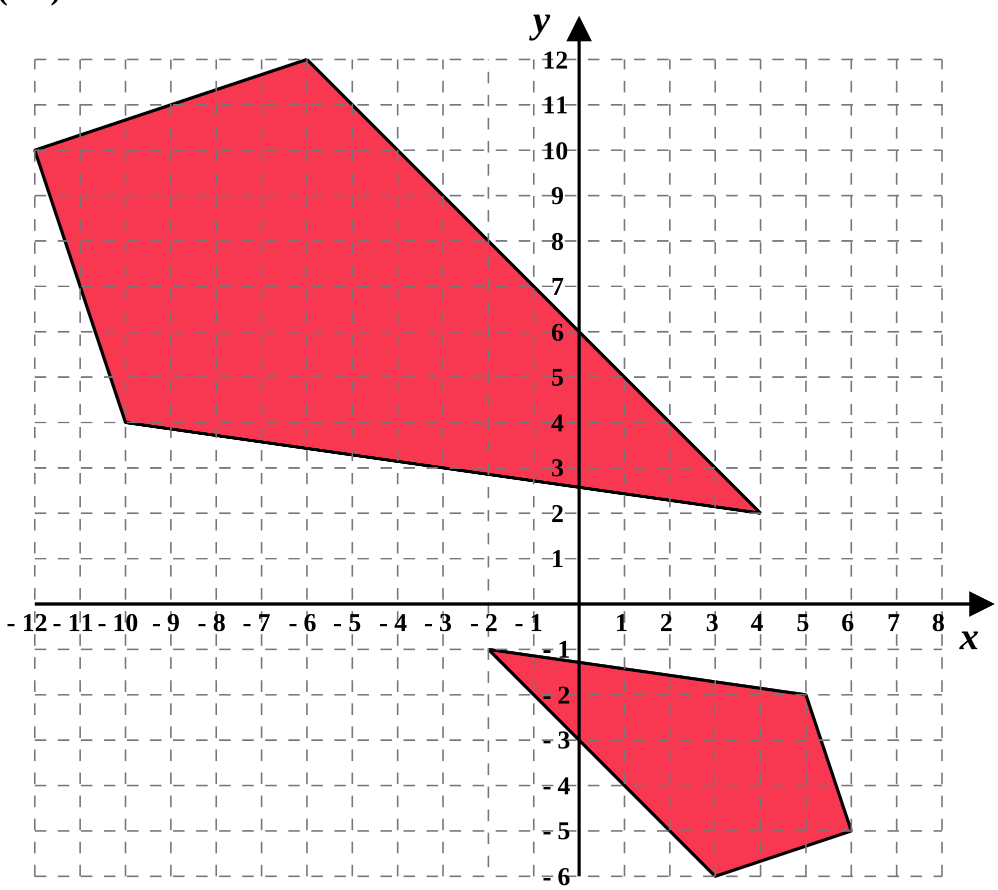
Answer 1

(i) and **(ii)**

$$\begin{array}{c|c} & \begin{pmatrix}5 & 6 & 3 & -2\\ -2 & -5 & -6 & -1\end{pmatrix}\\ \hline \begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix} & \begin{pmatrix}-10 & -12 & -6 & 4\\ 4 & 10 & 12 & 2\end{pmatrix}\end{array}$$

[1, 4 marks]

(iii)



[1 mark]

(iv)

$$\text{Area scale factor} = |\mathbf{M}|$$

$$= \left| \begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix} \right|$$

$$= (-2) \times (-2) - 0 \times 0$$

$$= 4$$

[1 mark]

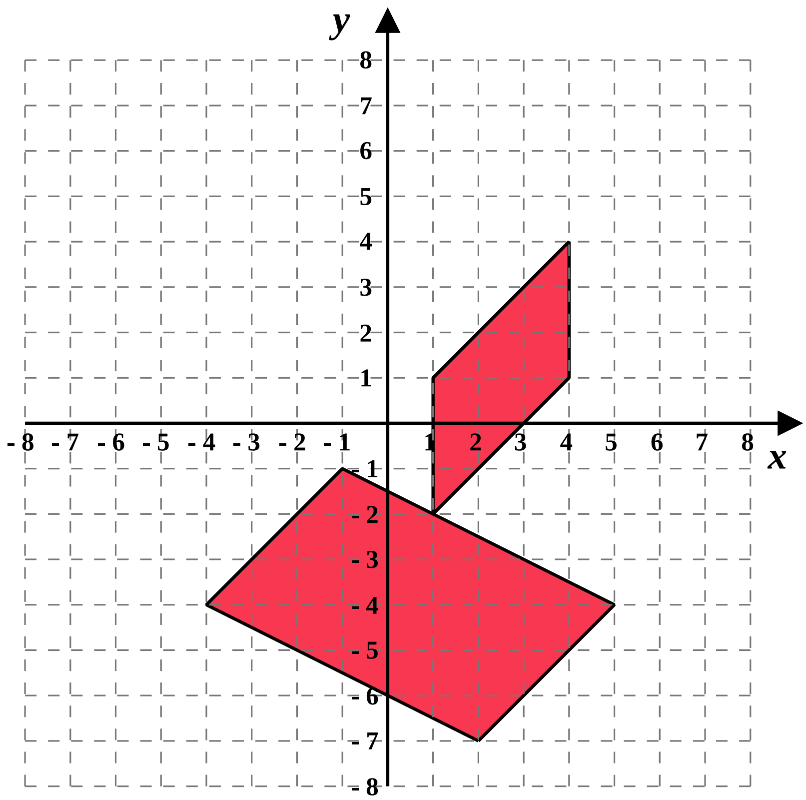
Answer 2

(i) and **(ii)**

	$\begin{pmatrix} 1 & 4 & 4 & 1 \\ -2 & 1 & 4 & 1 \end{pmatrix}$
$\begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$	$\begin{pmatrix} 5 & 2 & -4 & -1 \\ -4 & -7 & -4 & -1 \end{pmatrix}$

[1, 4 marks]

(iii)



[1 mark]

(iv)

$$\textit{det}\ \mathbf{M} = \left| \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix} \right|$$

$$= 1 \times 1 - (-2) \times (-2)$$

$$= -3$$

The magnitude of the determinant gives the area scale factor, greater than one, of the transformation.

(The minus sign means that the orientation of the shape has reversed)

[1 mark]

Answer 3

$$\begin{array}{c|c} & \begin{pmatrix} 2 & 9 & 1 & -6 \\ -3 & 12 & -5 & 7 \\ 4 & -2 & 0 & 11 \end{pmatrix} \\ \hline \begin{pmatrix} 5 & -1 & 6 \\ 8 & 3 & -4 \end{pmatrix} & \begin{pmatrix} 37 & 21 & 10 & 29 \\ -9 & 116 & -7 & -71 \end{pmatrix} \end{array}$$

Thus,

$$\begin{pmatrix} 5 & -1 & 6 \\ 8 & 3 & -4 \end{pmatrix} \times \begin{pmatrix} 2 & 9 & 1 & -6 \\ -3 & 12 & -5 & 7 \\ 4 & -2 & 0 & 11 \end{pmatrix} = \begin{pmatrix} 37 & 21 & 10 & 29 \\ -9 & 116 & -7 & -71 \end{pmatrix}$$

[5 marks]

Answer 4

$$\begin{aligned} det\,\mathbf{M} &= |\mathbf{M}| \\ &= \left| \begin{pmatrix} 2k + 5 & -4 \\ 1 & k \end{pmatrix} \right| \\ &= (2k + 5) \times k - (-4) \times 1 \\ &= 2k^2 + 5k + 4 \end{aligned}$$

$\therefore$ For $det\,\mathbf{M}$ to equal zero requires that $2k^2 + 5k + 4 = 0$

METHOD 1

Consider the discriminant, $D = 5^2 - 4 \times 2 \times 4$

$= -7$

As $D < 0$ there are no solutions to the equation $2k^2 + 5k + 4 = 0$

Thus, $det\,\mathbf{M} \neq 0$ for all values of k □

[4 marks]

METHOD 2

By completing the square,

$$2k^2 + 5k + 4 = 2\left(k + \frac{5}{4}\right)^2 + \frac{7}{8}$$

Anything squared has a minimum value of zero and, even then seven eighths is added on, so the quadratic in k is always positive.

Thus, $det\,\mathbf{M} \neq 0$ for all values of k □

[4 marks]

Answer 5

(i)

$$\begin{aligned} det\,\mathbf{M} &= \pm 2 \\ det\begin{pmatrix} -4 & 10 \\ -3 & k \end{pmatrix} &= \pm 2 \\ (-4) \times k - 10 \times (-3) &= \pm 2 \\ -4k + 30 &= \pm 2 \\ 4k - 30 &= \mp 2 \\ 4k &= \mp 2 + 30 \\ k &= \frac{30 \mp 2}{4} \\ k &= 7 \text{ or } 8 \end{aligned}$$

[4 marks]

(ii)

$$\begin{array}{c|c|} & \begin{pmatrix} 2 & 8 \\ 0 & 2 \\ 1 & -2 \end{pmatrix} \\ \hline \begin{pmatrix} 1 & -2 & 3 \\ -2 & 5 & 1 \end{pmatrix} & \begin{pmatrix} 5 & -2 \\ -3 & -8 \end{pmatrix} \\ \hline \end{array}$$

$$\therefore\ \mathbf{BC} = \begin{pmatrix} 5 & -2 \\ -3 & -8 \end{pmatrix}$$

[3 marks]