

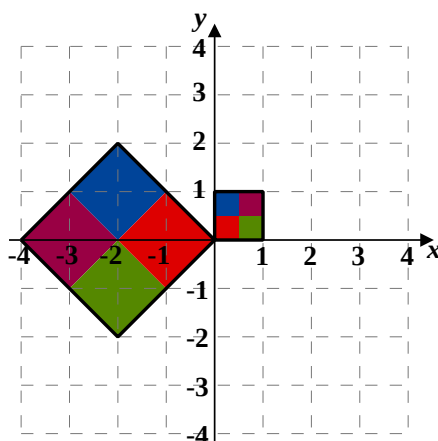
4.3 Answers

Further A-Level Pure Mathematics : Core 1 Matrix Transformations

Answer 1

(i)

$$\begin{array}{c|c} \text{NU} & \left(\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right) \\ \hline \left(\begin{array}{cc} -2 & -2 \\ -2 & 2 \end{array} \right) & \left(\begin{array}{cccc} 0 & -2 & -4 & -2 \\ 0 & -2 & 0 & 2 \end{array} \right) \end{array}$$



[3 marks]

(ii)

$$\begin{aligned} \det \mathbf{N} &= |\mathbf{N}| \\ &= \left| \begin{pmatrix} -2 & -2 \\ -2 & 2 \end{pmatrix} \right| \\ &= (-2) \times 2 - (-2) \times (-2) \\ &= -8 \end{aligned}$$

[1 mark]

- (iii) The minus sign of the determinant informs that the orientation of the shape has been reversed.
Going anticlockwise about the centre the colours used to go red, green purple, blue but in the image go red, blue, purple, green.

[2 marks]

(iv) $lsf = 2\sqrt{2}$

[1 mark]

(v) $asf = 8$

[1 mark]

Answer 2

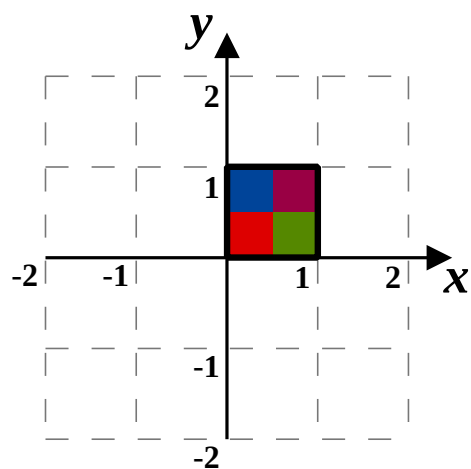
A Catalogue of Two-Dimensional Transformations

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Description : Identity matrix, **I**

Leaves points as is

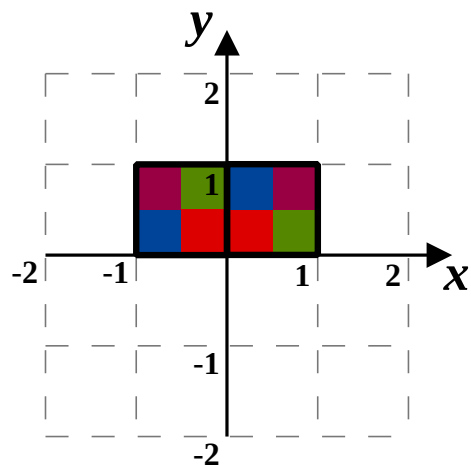


$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$

Description : Rotation of 90°

about $(0, 0)$

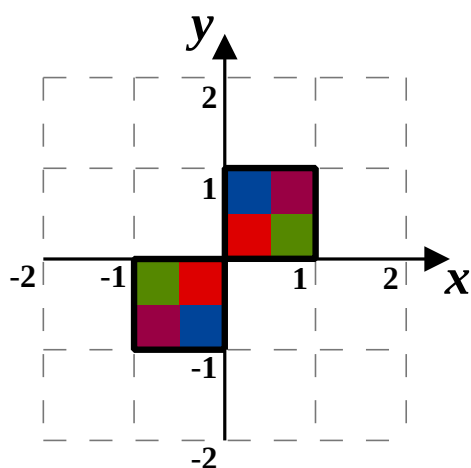


$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$

Description : Rotation of 180°

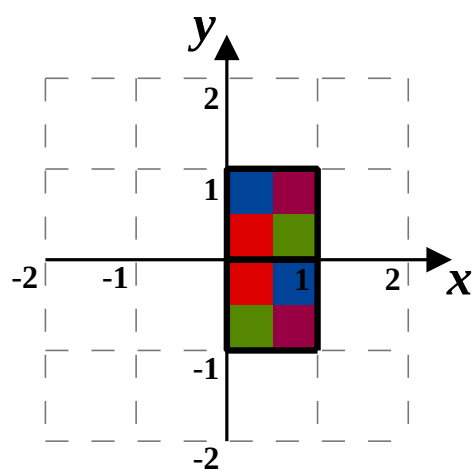
about $(0, 0)$



$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$

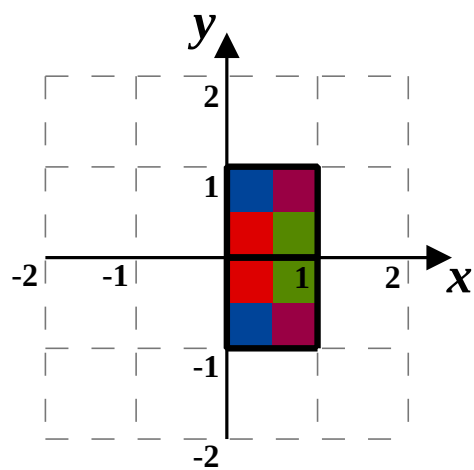
Description : Rotation of 270°
about $(0, 0)$



$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{pmatrix}$

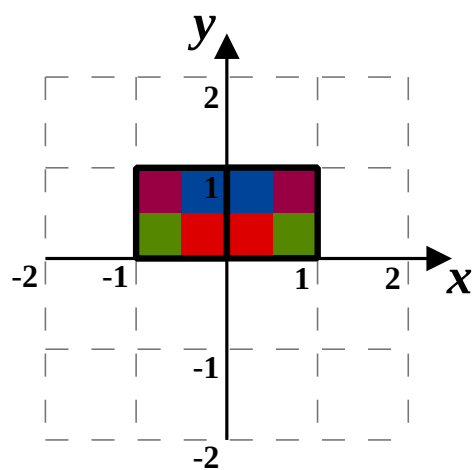
Description : Reflection in x-axis



$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

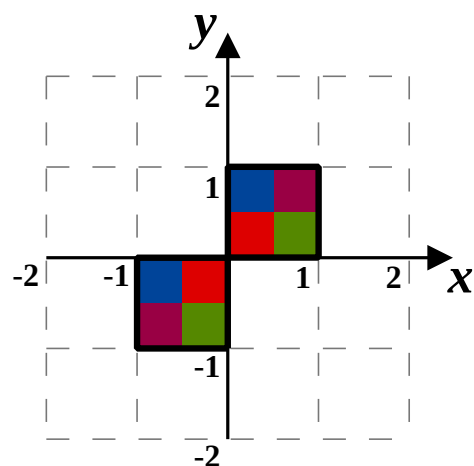
Description : Reflection in y-axis



$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & -1 & -1 & 0 \end{pmatrix}$

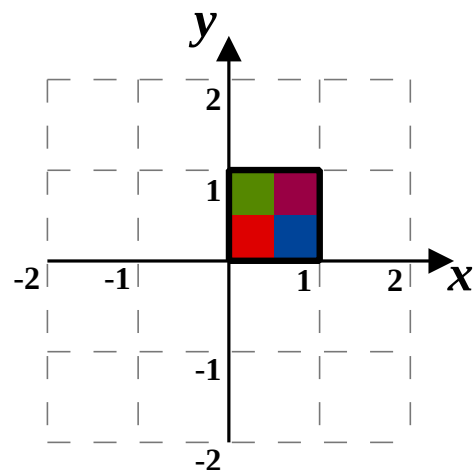
Description : Reflection in $y = -x$



$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$

Description : Reflection in $y = x$



[3 marks each = 24 marks total]

Answer 3

When a matrix is singular,

$$\det \mathbf{M} = 0$$

$$\left| \begin{pmatrix} 1+k & k \\ k & 1-k \end{pmatrix} \right| = 0$$

$$(1+k) \times (1-k) - k \times k = 0$$

$$1 - k^2 - k^2 = 0$$

$$2k^2 - 1 = 0$$

$$k^2 = \frac{1}{2}$$

$$k = \pm \frac{1}{\sqrt{2}}$$

$$\pm \frac{\sqrt{2}}{2}$$

[3 marks]

Answer 4**(a)**

$$\begin{array}{c|c} \mathbf{AB} & \left(\begin{array}{cc} -3 & -1 \\ 5 & 2 \end{array} \right) \\ \hline \left(\begin{array}{cc} 2 & 0 \\ 5 & 3 \end{array} \right) & \left(\begin{array}{cc} -6 & -2 \\ 0 & 1 \end{array} \right) \end{array}$$

$$\therefore \mathbf{AB} = \left(\begin{array}{cc} -6 & -2 \\ 0 & 1 \end{array} \right)$$

[3 marks]**(b)** C is Reflection in the y-axis**[2 marks]**

$$\mathbf{(c)} \quad \mathbf{C}^{100} = \mathbf{I} = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right)$$

[1 mark]**Answer 5****(a) (i)**

$$\begin{array}{c|c} \mathbf{A}^2 & \left(\begin{array}{cc} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{array} \right) \\ \hline \left(\begin{array}{cc} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{array} \right) & \left(\begin{array}{cc} 3 & 0 \\ 0 & 3 \end{array} \right) \end{array}$$

[3 marks]**(ii)** Enlargement, scale factor 3, centre (0, 0)**[1 mark]****(b)** Reflection in the line $y = -x$ **[2 marks]****(c)** When a matrix is singular, $\det \mathbf{C} = 0$

$$\det \mathbf{C} = 0$$

$$\left| \left(\begin{array}{cc} k+1 & 12 \\ k & 9 \end{array} \right) \right| = 0$$

$$(k+1) \times 9 - 12 \times k = 0$$

$$9 - 3k = 0$$

$$k = 3$$

[3 marks]