5.3 Answers to 5.2 Exercise

Answer 1

(i) Plotted as **A**

(ii)

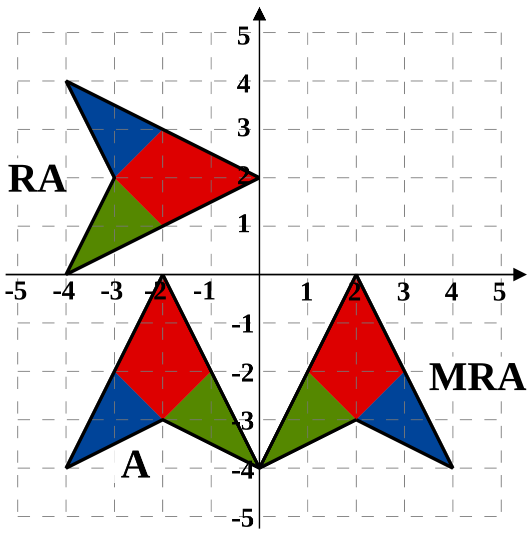
RA	$\begin{pmatrix} -2 & 0 & -2 & -4 \\ 0 & -4 & -3 & -4 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -4 & -3 & -4 \\ 2 & 0 & 2 & 4 \end{pmatrix}$

Plotted as **RA**

(iii)

MRA	$\begin{pmatrix} 0 & -4 & -3 & -4 \\ 2 & 0 & 2 & 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 & 0 & 2 & 4 \\ 0 & -4 & -3 & -4 \end{pmatrix}$

Plotted as **MRA**



(iv)

MR	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

This is recognisable as the matrix for reflection in the y-axis

The single matrix that would have transformed **A** to **MRA** is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

[2, 3, 3, 4 marks]

Answer 2

(i) Plotted as **P**

[2 marks]

(ii)

SP	$\begin{pmatrix} 0 & 2 & 0 & -2 \\ 0 & 0 & -4 & -4 \end{pmatrix}$
$\begin{pmatrix} -1 & 1 \\ 1 & -0.5 \end{pmatrix}$	$\begin{pmatrix} 0 & -2 & -4 & -2 \\ 0 & 2 & 2 & 0 \end{pmatrix}$

Plotted as **SP**

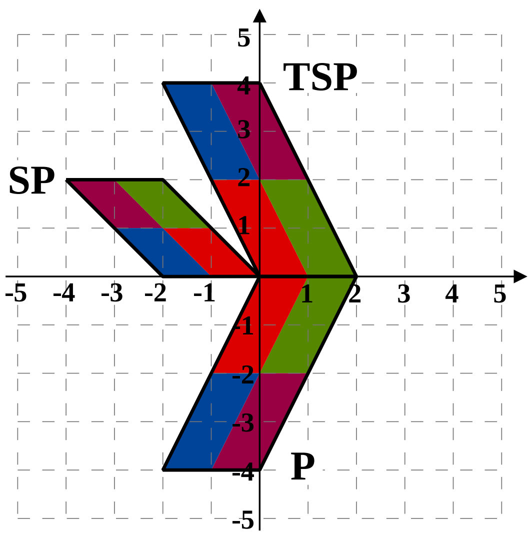
[3 marks]

(iii)

TSP	$\begin{pmatrix} 0 & -2 & -4 & -2 \\ 0 & 2 & 2 & 0 \end{pmatrix}$
$\begin{pmatrix} 1 & 2 \\ -2 & -2 \end{pmatrix}$	$\begin{pmatrix} 0 & 2 & 0 & -2 \\ 0 & 0 & 4 & 4 \end{pmatrix}$

Plotted as **TSP**

[3 marks]



(iv)

TS	$\begin{pmatrix} -1 & 1 \\ 1 & -0.5 \end{pmatrix}$
$\begin{pmatrix} 1 & 2 \\ -2 & -2 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

This is recognisable as the matrix for reflection in the *x*-axis

The single matrix that would have transformed **P** to **TSP** is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

[4 marks]

Answer 3

(a) $\mathbf{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(b) $\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(c) **R** = **QP**

(d)

$$\begin{array}{c|c} \mathbf{QP} & \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} & \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \end{array}$$

$$\mathbf{R} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(e) **R** is a reflection in the y-axis

This question is showing that a rotation 90° anticlockwise about the origin followed by a reflection in the line $y = -x$ is equivalent to a reflection in the in y-axis.

[1, 1, 1, 2, 2 marks]

Answer 4

(a)

$$\begin{array}{c|c} \mathbf{PA} & \begin{pmatrix} 1 & 2 & 2 \\ 1 & 1 & 4 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 1 & 1 & 4 \\ 1 & 2 & 2 \end{pmatrix} \end{array}$$

So T' has coordinates $A'(1, 1)$, $B'(1, 2)$ and $C'(4, 2)$

(b) **P** represents reflection in the line $y = x$

(c)

$$\begin{array}{c|c} \mathbf{QR} & \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \\ \hline \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix} & \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix} \end{array}$$

(d) $|\mathbf{QR}| = (-2) \times 2 - 0 \times 0 = -4$

(e) T has area $0.5 \times 1 \times 3 = 1.5$

$\therefore$ T'' has area $1.5 \times 4 = 6$ units^2

[2, 2, 2, 2, 3 marks]

Answer 5

(a) A "matrix methods solution"...

$$\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}\begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$$

$$\begin{pmatrix} (-4 \times 4 + a \times 6) \\ b \times 4 + (-2) \times 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$$

$$\begin{pmatrix} -16 + 6a \\ 4b - 12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$$

$$\begin{pmatrix} 6a \\ 4b \end{pmatrix} - \begin{pmatrix} 16 \\ 12 \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix}$$

$$\begin{pmatrix} 6a \\ 4b \end{pmatrix} = \begin{pmatrix} 2 \\ -8 \end{pmatrix} + \begin{pmatrix} 16 \\ 12 \end{pmatrix}$$

$$\begin{pmatrix} 6a \\ 4b \end{pmatrix} = \begin{pmatrix} 18 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

[4 marks]

(b)

$$\begin{aligned} det\,|\mathbf{A}| &= \left|\begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}\right| \\ &= \left|\begin{pmatrix} -4 & 3 \\ 1 & -2 \end{pmatrix}\right| \\ &= (-4) \times (-2) - 3 \times 1 \\ &= 8 - 3 \\ &= 5 \end{aligned}$$

$\therefore$ Area of S will be $30 \times 5 = 150$ square units

[4 marks]