6.3 Answers

Answer 1

(i) As $cos\ \theta$ and $sin\ \theta$ are both positive, $0 < \theta < 90$

$\therefore\ \theta = 30°$

Y represents a rotation of 30° about the origin.

[2 marks]

(ii)

$$\mathbf{YM} = \frac{1}{2}\begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix} \times 2 \times \begin{pmatrix} 2\sqrt{3} & 4\sqrt{3} & 4\sqrt{3} & 2\sqrt{3} \\ 1 & 1 & 3 & 3 \end{pmatrix}$$

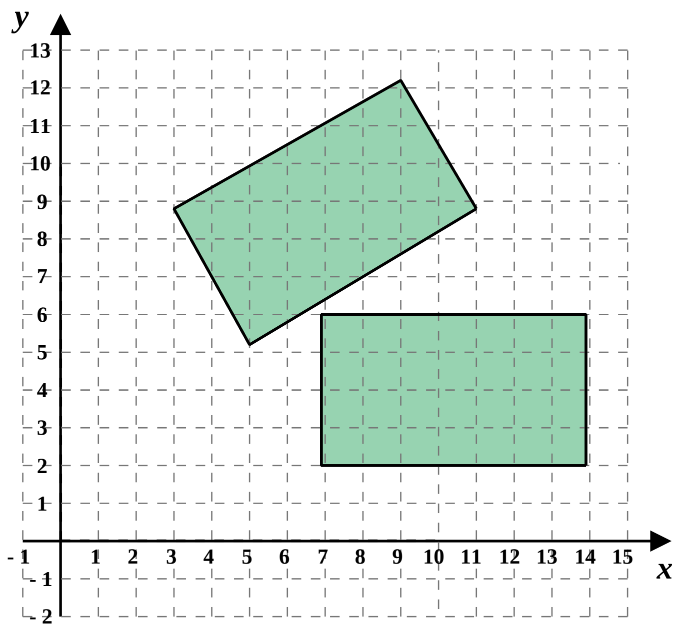
$$= \begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}\begin{pmatrix} 2\sqrt{3} & 4\sqrt{3} & 4\sqrt{3} & 2\sqrt{3} \\ 1 & 1 & 3 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 11 & 9 & 3 \\ 3\sqrt{3} & 5\sqrt{3} & 7\sqrt{3} & 5\sqrt{3} \end{pmatrix}$$

YM	$\begin{pmatrix} 2\sqrt{3} & 4\sqrt{3} & 4\sqrt{3} & 2\sqrt{3} \\ 1 & 1 & 3 & 3 \end{pmatrix}$
$\begin{pmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{pmatrix}$	$\begin{pmatrix} 5 & 11 & 9 & 3 \\ 3\sqrt{3} & 5\sqrt{3} & 7\sqrt{3} & 5\sqrt{3} \end{pmatrix}$

[5 marks]

(iii)



$$\text{YM} = \begin{pmatrix} 5 & 11 & 9 & 3 \\ 5.20 & 8.66 & 12.12 & 8.66 \end{pmatrix}$$

[2 marks]

Answer 2

(a) As *cos* θ and *sin* θ are both positive, $0 < \theta < 90$

$\therefore\ \theta = 45°$

M represents a rotation of 45° about the origin.

[2 marks]

(b)

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \sqrt{2} \times \begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} p - q \\ p + q \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

Simultaneous Equations

$$2p = 14$$

$$p = 7 \qquad \Rightarrow q = 1$$

[4 marks]

(c)

$$\begin{aligned} |OA| &= \sqrt{p^2 + q^2} \\ &= \sqrt{7^2 + 1^2} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \end{aligned}$$

[2 marks]

(d)

$$\begin{aligned} \mathbf{M}^2 &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \\ &= \frac{1}{2} \times \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \end{aligned}$$

Rotation 90° about (0, 0)

[2 marks]

(e)

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3\sqrt{2} \\ 4\sqrt{2} \end{pmatrix} = \begin{pmatrix} -4\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$$

$$\therefore\ C\left(-4\sqrt{2},\ 3\sqrt{2}\right)$$

[2 marks]

Answer 3

(a) As $cos\ \theta$ is negative and $sin\ \theta$ is positive positive, $90 < \theta < 180$

$\therefore\ \theta = 135°$

P represents a rotation of 135° about the origin.

[2 marks]

(b)

$$\frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 6\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \sqrt{2} \times \begin{pmatrix} 6\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} -p - q \\ p - q \end{pmatrix} = \begin{pmatrix} 12 \\ 6 \end{pmatrix}$$

Simultaneous Equations

$$-2q = 18$$

$$q = -9 \qquad \Rightarrow\ p = -3$$

[3 marks]

(c)

$$\mathbf{Q} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

[1 mark]

(d)

$$\mathbf{R} = \mathbf{QP}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \times \frac{1}{\sqrt{2}} \times \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$$

[3 marks]

(e) If **R** is self inverse then **RR** = **I**, the identity matrix

$$\mathbf{RR} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \times \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \mathbf{I} \qquad \therefore\ \mathbf{R} \text{ is self inverse}$$

It can also be thought of as a reflection.
I think the mirror line is $y = -\left(1 + \sqrt{2}\right) x$
Any reflection twice in a line gets back to the original position.

[2 marks]

Answer 4

(i) **(a)** Reflection in the line $y = -x$: $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

(b) Put 135° into the general rotation matrix : $\begin{pmatrix} \cos 135° & -\sin 135° \\ \sin 135° & \cos 135° \end{pmatrix}$

$$\begin{pmatrix} \dfrac{-1}{\sqrt{2}} & \dfrac{-1}{\sqrt{2}} \\[2ex] \dfrac{1}{\sqrt{2}} & \dfrac{-1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$$

(c) Get the order correct...

$$\frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

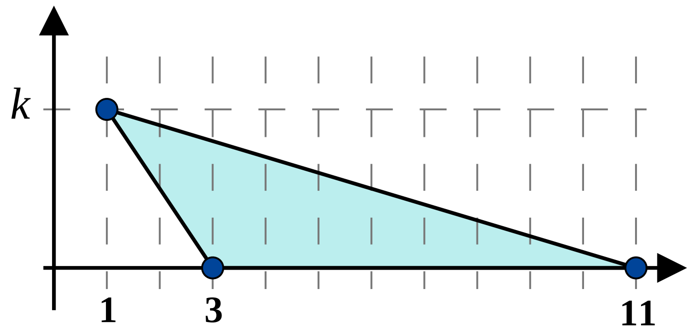
$$= \begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\[2ex] \dfrac{1}{\sqrt{2}} & \dfrac{-1}{\sqrt{2}} \end{pmatrix}$$

[4 marks]

(ii) $det\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix} = 6 \times 2 - (-2) \times 1 = 14$

So the area of triangle T must be $\dfrac{364}{14} = 26$ square units

A quick sketch...



$$\text{Area } \Delta = \frac{1}{2}\, b\, h$$

$$26 = \frac{1}{2} \times 8 \times k$$

$$k = 6.5 \text{ units}$$

[6 marks]

Answer 5

(i)

$$\mathbf{P}^2 = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \times \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta - \sin^2\theta & -\cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta + \sin\theta\cos\theta & -\sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

[6 marks]

(ii) Two successive anticlockwise rotations about the origin by an angle θ are equivalent to a single anticlockwise rotation by an angle 2θ

[2 marks]