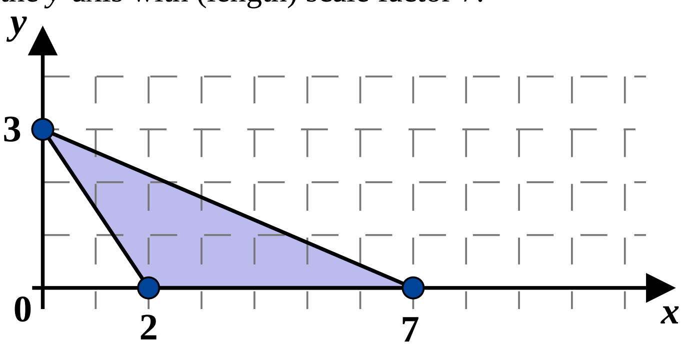
Answer 1

(i) The matrix is of the form $\begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}$ and so $\begin{pmatrix} 5 & 0 \\ 0 & 7 \end{pmatrix}$ represents a stretch parallel to the x-axis, with (length) scale factor 5 and a stretch parallel to the y-axis with (length) scale factor 7.

(ii)



$$\text{Area } \Delta = \frac{1}{2}bh = \frac{1}{2} \times 5 \times 3 = 7.5 \text{ units}^2$$

(iii) $det \begin{pmatrix} 5 & 0 \\ 0 & 7 \end{pmatrix} = 5 \times 7 - 0 \times 0 = 35$ (which is the transformation's *asf*)

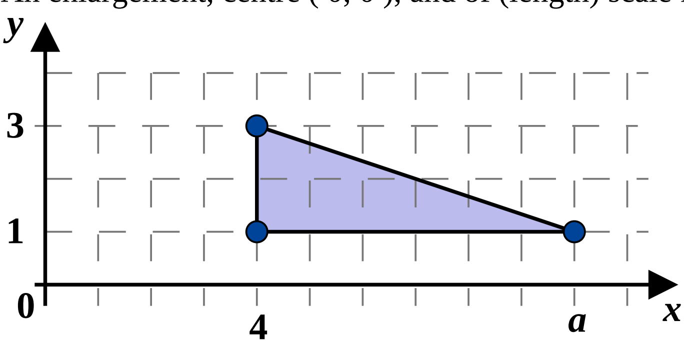
$\therefore$ Area of image of $T = 35 \times 7.5 = 262.5$ units^2

[3, 3, 3 marks]

Answer 2

(i) An enlargement, centre (0, 0), and of (length) scale factor $2\sqrt{5}$

(ii)



$det \begin{pmatrix} 2\sqrt{5} & 0 \\ 0 & 2\sqrt{5} \end{pmatrix} = 2\sqrt{5} \times 2\sqrt{5} - 0 \times 0 = 20$

This is the Area Scale Factor of the transformation

$\therefore$ Area of of $T = 3$ units^2

$$\text{Area } \Delta = \frac{1}{2}bh$$

$$3 = \frac{1}{2} \times base \times 2$$

$$base = 3 \text{ units} \qquad \Rightarrow a = 1 \text{ or } 7$$

[3, 4 marks]

Answer 3

(a) **(i)**

$\mathbf{A}^2$	$\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$
$\begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

(ii) Enlargement, centre (0, 0), (length) scale factor 3

[4 marks]

(b) Reflection in the line $y = -x$

[2 marks]

(c) A singular matrix has a determinant of zero,

$$det\,\mathbf{C} = det\begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix}$$

$$0 = (k+1) \times 9 - 12k$$

$$0 = 9k + 9 - 12k$$

$$3k = 9 \Leftrightarrow k = 3$$

[3 marks]

Answer 4

(a) $\begin{pmatrix} \dfrac{1}{\sqrt{2}} & \dfrac{-1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \end{pmatrix}$

(b) Enlargement, centre (0, 0), (length) scale factor $k\sqrt{2}$

(c) $\mathbf{PQ} = \dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} k\sqrt{2} & 0 \\ 0 & k\sqrt{2} \end{pmatrix}$

$$= \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$$

$$= \begin{pmatrix} k & -k \\ k & k \end{pmatrix} \quad \text{or} \quad k\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

The Area Scale Factor of the transformation, also *det* (**PQ**) will be,

$$\frac{\text{Area } R_2}{\text{Area } R_1} = \frac{147}{6} = \frac{49}{2}$$

$$det\,(\mathbf{PQ}) = k \times k - (-k) \times k = 2k^2$$

$$\therefore\ k = \sqrt{\frac{49}{4}} = \frac{7}{2} = 3.5$$

[1, 2, 4 marks]

Answer 5

(a) $\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = \mathbf{E}$

[1 mark]

(b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \mathbf{R}$

[1 mark]

(c) **T** = **RE**

RE	$\begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix}$

$$\therefore\ \mathbf{T} = \begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix}$$

[2 marks]

(d)

AB	$\begin{pmatrix} k & 1 \\ c & -6 \end{pmatrix}$
$\begin{pmatrix} 6 & 1 \\ 4 & 2 \end{pmatrix}$	$\begin{pmatrix} 6k + c & 0 \\ 4k + 2c & -8 \end{pmatrix}$

$$\therefore\ \mathbf{AB} = \begin{pmatrix} 6k + c & 0 \\ 4k + 2c & -8 \end{pmatrix}$$

[3 marks]

(e)

$$\begin{pmatrix} 8 & 0 \\ 0 & -8 \end{pmatrix} = \begin{pmatrix} 6k + c & 0 \\ 4k + 2c & -8 \end{pmatrix}$$

$$6k + c = 8$$

$$4k + 2c = 0$$

$$12k + 2c = 16$$

$$4k + 2c = 0 \qquad \text{SUBTRACT}$$

$$8k = 16$$

$$k = 2 \quad \Rightarrow \quad c = -4$$

[2 marks]

Answer 6

(a)

$$\begin{aligned}
\text{LHS} &= \mathbf{P}^3 \\
&= \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \\
&= \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} \\
&= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} \\
&= 8\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&= 8\mathbf{I} \\
&= \text{RHS} \qquad \square
\end{aligned}$$

[3 marks]

(b) It's a spiral similarity which can be broken down into,

- An enlargement, centre (0, 0), and (length) scale factor 2
- A rotation of 120° about (0, 0)

In this case the order will not matter.

[4 marks]

(c)

$$\begin{aligned}
\mathbf{P}^{35} &= \left(\mathbf{P}^3\right)^{11} \times \mathbf{P}^2 \\
&= (8\mathbf{I})^{11} \times \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} \\
&= 8^{11} \times \mathbf{I}^{11} \times 2 \times \begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix} \\
&= \left(2^3\right)^{11} \times 1^{11} \times 2 \times \begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix} \\
&= 2^{34}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}
\end{aligned}$$

$$\therefore \quad k = 34, \quad a = \sqrt{3} \text{ and } b = -\sqrt{3}$$

[2 marks]