8.4 Answers

Answer 1

(i)

$$\begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} k \\ k+1 \end{pmatrix} = \begin{pmatrix} k - (k+1) \\ -2k + 2(k+1) \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

[2 marks]

(ii) $(-1,\ 2)$

[1 mark]

(iii) A generalised point on the line $y = x - 2$ would $(k,\ k-2)$

$$\begin{pmatrix} 1 & -1 \\ -2 & 2 \end{pmatrix}\begin{pmatrix} k \\ k-2 \end{pmatrix} = \begin{pmatrix} k - (k-2) \\ -2k + 2(k-2) \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

All points on the line $y = x - 2$ are mapped onto $(2, -4)$

[3 marks]

Answer 2

(a) $det\,\mathbf{M} = 1 \times 1 - (-3) \times 2 = 7$

[1 mark]

(b)

$$\begin{aligned} det\,\mathbf{N} &= 7 \\ (-1) \times 3 - 4k &= 7 \\ -4k &= 10 \\ k &= -2.5 \end{aligned}$$

[1 mark]

(c)

MN	$\begin{pmatrix} -1 & -2.5 \\ 4 & 3 \end{pmatrix}$
$\begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix}$	$\begin{pmatrix} -13 & -11.5 \\ 2 & -2 \end{pmatrix}$

[1 mark]

(d)

$$\begin{aligned} \text{LHS} &= det\,\mathbf{MN} \\ &= (-13) \times (-2) - (-11.5) \times 2 \\ &= 26 + 23 \\ &= 49 \\ &= 7 \times 7 \\ &= det\,\mathbf{M} \times det\,\mathbf{N} \\ &= \text{RHS} \qquad \square \end{aligned}$$

[1 mark]

(e)

$$\begin{aligned} det\,(\mathbf{M} + \mathbf{N}) &= det\left(\begin{pmatrix} 1 & -3 \\ 2 & 1 \end{pmatrix} + \begin{pmatrix} -1 & -2.5 \\ 4 & 3 \end{pmatrix} \right) \\ &= det \begin{pmatrix} 0 & -5.5 \\ 6 & 4 \end{pmatrix} \\ &= 0 \times 4 - (-5.5) \times 6 \\ &= 33 \end{aligned}$$

$$\begin{aligned} det\,\mathbf{M} + det\,\mathbf{N} &= 7 + 7 \\ &= 14 \end{aligned}$$

$$\therefore \; det\,(\mathbf{M} + \mathbf{N}) \neq det\,\mathbf{M} + det\,\mathbf{N}$$

[2 marks]

Answer 3

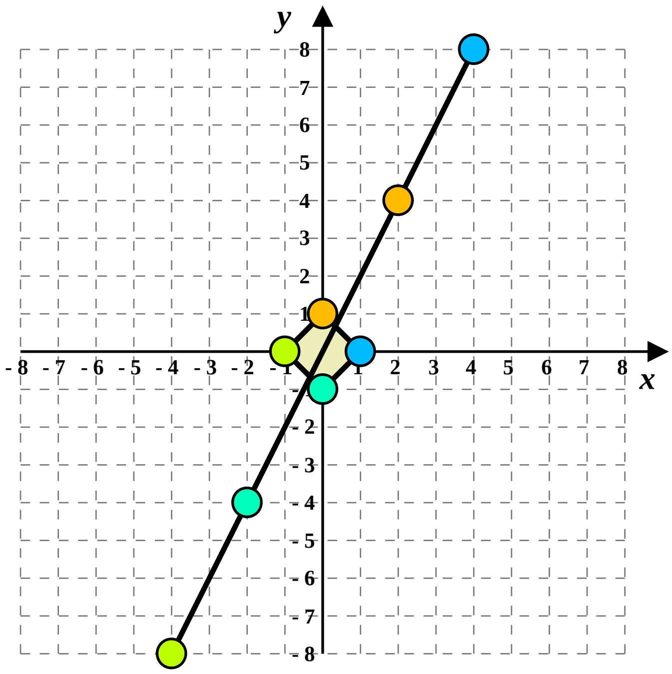
(i) $\quad \mathit{det}\ \mathbf{A} = 24 \qquad \mathit{det}\ \mathbf{B} = 0 \qquad \mathit{det}\ \mathbf{C} = -4$

$\therefore$ **B** is singular

[2 marks]

(ii)

M□	$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$
$\begin{pmatrix} 4 & 2 \\ 8 & 4 \end{pmatrix}$	$\begin{pmatrix} 4 & 2 & -4 & -2 \\ 8 & 4 & -8 & -4 \end{pmatrix}$



[3 marks]

(iii) $\quad y = 2x$

[1 mark]

(iv) For a general pre-image point (u, v), and image point (x, y)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 8 & 4 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix}$$

$$= \begin{pmatrix} 4u + 2v \\ 8u + 4v \end{pmatrix}$$

$$= \begin{pmatrix} 4u + 2v\,) \\ 2\,(\,4u + 2v\,) \end{pmatrix}$$

Let $p = 4u + 2v$ to get the result that,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ 2p \end{pmatrix}$$

Which are points on the line $y = 2x$

All points under the transformation represented by this matrix, have images on the line $y = 2x$.

The two dimensional surface is collapsing into a one dimensional line !

[3 marks]

(v) Let a general point on the line $y = -2x + 3$ be $(k, -2k + 3)$

$$\begin{pmatrix} 4 & 2 \\ 8 & 4 \end{pmatrix}\begin{pmatrix} k \\ -2k + 3 \end{pmatrix} = \begin{pmatrix} 4k + 2(-2k + 3) \\ 8k + 4(-2k + 3) \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 12 \end{pmatrix}$$

So, all points of the line $y = -2x + 3$ map onto the point $(6, 12)$

[3 marks]

(vi) Let a general point on the line $y = -2x + c$ be $(k, -2k + c)$

$$\begin{pmatrix} 4 & 2 \\ 8 & 4 \end{pmatrix}\begin{pmatrix} k \\ -2k + c \end{pmatrix} = \begin{pmatrix} 4k + 2(-2k + c) \\ 8k + 4(-2k + c) \end{pmatrix}$$

$$= \begin{pmatrix} 2c \\ 4c \end{pmatrix}$$

So, all points of the line $y = -2x + c$ map onto the point $(2c, 4c)$

Question asks this be done for a value of c chosen by the candidate.

[3 marks]

Answer 4

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

[5 marks]

Answer 5

For $\mathbf{M}$ to be singular, $\quad det\,\mathbf{M} = 0$

$$(-2) \times k - (1 - k)(k - 1) = 0$$

$$-2k - \left(k - 1 - k^2 + k\right) = 0$$

$$-2k - k + 1 + k^2 - k = 0$$

$$k^2 - 4k + 1 = 0$$

$$\left[k^2 - 4k\right] + 1 = 0$$

$$\left[(k - 2)^2 - 4\right] + 1 = 0$$

$$(k - 2)^2 - 3 = 0$$

$$(k - 2)^2 = 3$$

$$k - 2 = \pm\sqrt{3}$$

$$k = 2 \pm \sqrt{3}$$

[3 marks]

Answer 6

$$\text{LHS} = det\,\mathbf{MN}$$

$$= det\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix}\right)$$

$$= det\begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$$

$$= (ae + bg)(cf + dh) - (af + bh)(ce + dg)$$

$$= aecf + aedh + bgcf + bgdh - afce - afdg - bhce - bhdg$$

$$= (aecf - afce) + aedh + bgcf + (bgdh - bhdg) - afdg - bhce$$

$$= aedh + bgcf - afdg - bhce$$

$$\textbf{RHS} = det\,\mathbf{M} \times det\,\mathbf{N}$$

$$= det\begin{pmatrix} a & b \\ c & d \end{pmatrix} \times det\begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$= (ad - bc)(eh - fg)$$

$$= adeh + bcfg - adfg - bceh$$

$$= \textbf{LHS} \qquad \square$$

[5 marks]