2.3 Answers

2.3.1 Solutions to 2.1 Teaching Video Example

Rearrange the equation of the quadratic,

$$y = x^2 - 13$$
$$x^2 = y + 13$$

Then substitute this in place of the x^2 in the equation of the circle,

$$(x^2) + y^2 = 25$$
$$(y + 13) + y^2 = 25$$
$$y^2 - y - 12 = 0$$
$$(y + 4)(y - 3) = 0$$
$$\text{Either} \quad y + 4 = 0 \quad \text{or} \quad y - 3 = 0$$
$$y = -4 \quad \text{or} \quad y = 3$$

Back with the equation for the quadratic,

$$x^2 = y + 13$$

square root both sides

$$x = \pm\sqrt{y + 13}$$
$$\text{So when } y = -4, \quad x = \pm\sqrt{-4 + 13} \Rightarrow x = \pm 3$$
$$\text{And when } y = 3, \quad x = \pm\sqrt{3 + 13} \Rightarrow x = \pm 4$$

The four points of intersection are,

$$(-3, -4), \ (3, -4), (-4, \ 3) \text{ and } (4, 3)$$

2.3.2 Solutions to 2.2 Exercise

Answer 1

Rearrange the equation of the quadratic,

$$y = \frac{x^2 - 37}{7}$$

$$7y = x^2 - 37$$

$$x^2 = 7y + 37$$

Then substitute this in place of the x^2 in the equation of the circle,

$$\left(x^2 \right) + y^2 = 25$$

$$\left(7y + 37 \right) + y^2 = 25$$

$$y^2 + 7y + 12 = 0$$

$$(y + 4)(y + 3) = 0$$

$$\text{Either} \quad y + 4 = 0 \quad \text{or} \quad y + 3 = 0$$

$$y = -4 \quad \text{or} \quad y = -3$$

Back with the equation for the quadratic,

$$x^2 = 7y + 37$$

square root both sides

$$x = \pm\sqrt{7y + 37}$$

$$\text{So when } y = -4, \quad x = \pm\sqrt{7 \times (-4) + 37} \Rightarrow x = \pm 3$$

$$\text{And when } y = -3, \quad x = \pm\sqrt{7 \times (-3) + 37} \Rightarrow x = \pm 4$$

The four points of intersection are,

$$(-3, -4), \quad (3, -4), (-4, -3) \text{ and } (4, -3)$$

Answer 2

Rearrange the equation of the quadratic,

$$y = \frac{x^2 - 41}{8}$$

$$8y = x^2 - 41$$

$$x^2 = 8y + 41$$

Then substitute this in place of the x^2 in the equation of the circle,

$$\left(x^2 \right) + y^2 = 25$$

$$\left(8y + 41 \right) + y^2 = 25$$

$$y^2 + 8y + 16 = 0$$

$$(y + 4)(y + 4) = 0$$

$$\text{Either} \quad y + 4 = 0 \quad \text{or} \quad y + 4 = 0$$

$$y = -4$$

Back with the equation for the quadratic,

$$x^2 = 8y + 41$$

square root both sides

$$x = \pm\sqrt{8y + 41}$$

$$\text{So when } y = -4, \quad x = \pm\sqrt{8 \times (-4) + 41} \Rightarrow x = \pm 3$$

The two points of intersection are,

$$(-3, -4), \quad (3, -4)$$

Answer 3

(i) Rearrange the equation of the quadratic,

$$y = \frac{x^2 - 40}{8}$$

$$8y = x^2 - 40$$

$$x^2 = 8y + 40$$

Then substitute this in place of the x^2 in the equation of the circle,

$$\left(x^2 \right) + y^2 = 25$$

$$\left(8y + 40 \right) + y^2 = 25$$

$$y^2 + 8y + 15 = 0$$

$$(y + 3)(y + 5) = 0$$

Either $y + 3 = 0$ or $y + 5 = 0$

$$y = -3 \quad \text{or} \quad y = -5$$

Back with the equation for the quadratic,

$$x^2 = 8y + 40$$

square root both sides

$$x = \pm\sqrt{8y + 40}$$

So when $y = -3, \quad x = \pm\sqrt{8 \times (-3) + 40} \Rightarrow x = \pm 4$

And when $y = -5, \quad x = \pm\sqrt{8 \times (-5) + 40} \Rightarrow x = 0$

The three points of intersection are,

$$(-4, -3), \quad (4, -3) \quad \text{and} \quad (0, -5)$$

(ii) Sophia's idea cannot be made to work.
The maximum possible number of points of intersection is four.
(Could be proved using The Fundamental Theorem of Algebra)

Answer 4

Rearrange the equation of the quadratic,

$$y = x^2 + 5$$

$$x^2 = y - 5$$

Then substitute this in place of the x^2 in the equation of the circle,

$$\left(x^2 \right) + y^2 = 25$$

$$\left(y - 5 \right) + y^2 = 25$$

$$y^2 + y - 30 = 0$$

$$(y + 6)(y - 5) = 0$$

$$\text{Either} \quad y + 6 = 0 \quad \text{or} \quad y - 5 = 0$$

$$y = -6 \quad \text{or} \quad y = 5$$

Back with the equation for the quadratic,

$$x^2 = y - 5$$

square root both sides

$$x = \pm\sqrt{y - 5}$$

$$\text{So when } y = -6, \quad x = \pm\sqrt{(-6) - 5} \Rightarrow x = \text{No real solutions}$$

$$\text{And when } y = 5, \quad x = \pm\sqrt{5 - 5} \Rightarrow x = 0$$

The only points of intersection is,

$$(0, \ 5)$$