3.4 Answers

GCSE and Preparatory A-Level Mathematics
Conic Sections (Simultaneous Equations IV)

3.4.1 Solution to 3.2 Teaching Video Example

Square both sides of the equation of the hyperbola,

$$y = \frac{12}{x} \Rightarrow y^2 = \frac{144}{x^2}$$

Then substitute this in place of the y^2 in the equation for the circle,

$$x^2 + \left(\frac{144}{x^2}\right) = 40$$

Multiply through by x^2

$$x^2 \times x^2 + \frac{x^2}{1} \times \left(\frac{144}{x^2}\right) = x^2 \times 40$$

$$x^4 + 144 = 40\,x^2$$

$$x^4 - 40\,x^2 + 144 = 0$$

$$\text{Let } z = x^2 \Rightarrow z^2 = x^4$$

$$z^2 - 40\,z + 144 = 0$$

$$(z - 4)(z - 36) = 0$$

$$\text{Either } \quad z - 4 = 0 \quad \text{or} \quad z - 36 = 0$$

$$z = 4 \quad \text{or} \quad z = 36$$

$$\therefore \; x^2 = 4 \text{ or } x^2 = 36$$

$$x = \pm 2 \quad \text{or} \quad x = \pm 6$$

Use the equation for the hyperbola to get the y part of the points,

$$y = \frac{12}{x}$$

The four points of intersection are,

$$(-2, -6), \; (2, \; 6), (-6, -2) \text{ and } \; (6, \; 2)$$

3.4.2 Solution to 3.3 Exercise

Answer 1

Square both sides of the equation of the hyperbola,

$$y = \frac{12}{x} \Rightarrow y^2 = \frac{144}{x^2}$$

Then substitute this in place of the y^2 in the equation for the circle,

$$x^2 + \left(\frac{144}{x^2} \right) = 25$$

Multiply through by x^2

$$x^2 \times x^2 + \frac{x^2}{1} \times \left(\frac{144}{x^2} \right) = x^2 \times 25$$

$$x^4 + 144 = 25x^2$$

$$x^4 - 25x^2 + 144 = 0$$

$$\text{Let } z = x^2 \Rightarrow z^2 = x^4$$

$$z^2 - 25z + 144 = 0$$

$$(z - 9)(z - 16) = 0$$

$$\text{Either } \quad z - 9 = 0 \quad \text{or} \quad z - 16 = 0$$

$$z = 9 \quad \text{or} \quad z = 16$$

$$\therefore \ x^2 = 9 \text{ or } x^2 = 16$$

$$x = \pm 3 \quad \text{or} \quad x = \pm 4$$

Use the equation for the hyperbola to get the y part of the points,

$$y = \frac{12}{x}$$

The four points of intersection are,

$$(-3, -4), \ (3, \ 4), (-4, -3) \text{ and } \ (4, \ 3)$$

Answer 2

Square both sides of the equation of the hyperbola,

$$y = \frac{9}{x} \Rightarrow y^2 = \frac{81}{x^2}$$

Then substitute this in place of the y^2 in the equation for the circle,

$$x^2 + \left(\frac{81}{x^2} \right) = 18$$

Multiply through by x^2

$$x^2 \times x^2 + \frac{x^2}{1} \times \left(\frac{81}{x^2} \right) = x^2 \times 18$$

$$x^4 + 81 = 18x^2$$

$$x^4 - 18x^2 + 81 = 0$$

$$\text{Let } z = x^2 \Rightarrow z^2 = x^4$$

$$z^2 - 18z + 81 = 0$$

$$(z - 9)(z - 9) = 0$$

$$\therefore \; z - 9 = 0$$

$$z = 9$$

$$\therefore \; x^2 = 9$$

$$x = \pm 3$$

Use the equation for the hyperbola to get the y part of the points,

$$y = \frac{9}{x}$$

The two points of intersection are,

$$(-3, -3), \; (3, \; 3)$$

Oscar has succeeded in his mission !

Answer 3

Square both sides of the equation of the hyperbola,

$$y = \frac{\sqrt{7}}{x} \Rightarrow y^2 = \frac{7}{x^2}$$

Then substitute this in place of the y^2 in the equation for the circle,

$$x^2 + \left(\frac{7}{x^2}\right) = 8$$

Multiply through by x^2

$$x^2 \times x^2 + \frac{x^2}{1} \times \left(\frac{7}{x^2}\right) = x^2 \times 8$$

$$x^4 + 7 = 8x^2$$

$$x^4 - 8x^2 + 7 = 0$$

$$\text{Let } z = x^2 \Rightarrow z^2 = x^4$$

$$z^2 - 8z + 7 = 0$$

$$(z - 1)(z - 7) = 0$$

$$\text{Either } \quad z - 1 = 0 \quad \text{or} \quad z - 7 = 0$$

$$z = 1 \quad \text{or} \quad z = 7$$

$$\therefore \ x^2 = 1 \text{ or } x^2 = 7$$

$$x = \pm 1 \quad \text{or} \quad x = \pm\sqrt{7}$$

Use the equation for the hyperbola to get the y part of the points,

$$y = \frac{\sqrt{7}}{x}$$

The four points of intersection are,

$$\left(-1, -\sqrt{7}\right), \left(1, \sqrt{7}\right), \left(-\sqrt{7}, -1\right) \text{ and } \left(\sqrt{7}, 1\right)$$

Answer 4

Square both sides of the equation of the hyperbola,

$$y = \frac{4}{x} \Rightarrow y^2 = \frac{16}{x^2}$$

Then substitute this in place of the y^2 in the equation for the circle,

$$x^2 + \left(\frac{16}{x^2}\right) = 10$$

Multiply through by x^2

$$x^2 \times x^2 + \frac{x^2}{1} \times \left(\frac{16}{x^2}\right) = x^2 \times 10$$

$$x^4 + 16 = 10x^2$$

$$x^4 - 10x^2 + 16 = 0$$

$$\text{Let } z = x^2 \Rightarrow z^2 = x^4$$

$$z^2 - 10z + 16 = 0$$

$$(z - 2)(z - 8) = 0$$

$$\text{Either } \quad z - 2 = 0 \quad \text{or} \quad z - 8 = 0$$

$$z = 2 \quad \text{or} \quad z = 8$$

$$\therefore \ x^2 = 2 \text{ or } x^2 = 8$$

$$x = \pm\sqrt{2} \quad \text{or} \quad x = \pm\sqrt{8} \qquad \text{Note } \sqrt{8} = 2\sqrt{2}$$

Use the equation for the hyperbola to get the y part of the points,

$$y = \frac{4}{x}$$

The four points of intersection are,

$$\left(-\sqrt{2}, -2\sqrt{2}\right), \left(\sqrt{2}, 2\sqrt{2}\right), \left(-2\sqrt{2}, -\sqrt{2}\right) \text{ and } \left(2\sqrt{2}, \sqrt{2}\right)$$