4.4 Answers

4.4.1 Solution to 4.2 Teaching Video Example

$$\begin{aligned} x^2 + 3y^2 &= 52 \\ x^2 + y^2 &= 34 \qquad \text{SUBTRACT} \end{aligned}$$

$$\begin{aligned} 2y^2 &= 18 \\ y^2 &= 9 \\ y &= \pm 3 \end{aligned}$$

From the graph, four points are expected, so at this stage they are shaping up to be,

$$(\quad, -3), \ (\quad, -3), \ (\quad, 3) \text{ and } (\quad, 3)$$

The x part of these coordinates are easiest to find from the equation of the circle,

$$\begin{aligned} x^2 + y^2 &= 34 \\ x^2 &= 34 - y^2 \\ &= 34 - (\pm 3)^2 \\ &= 34 - 9 \\ &= 25 \\ \therefore x &= \pm 5 \end{aligned}$$

The points are,

$$(-5, -3), \ (5, -3), \ (-5, 3) \text{ and } (5, 3)$$

4.4.2 Solution to 4.3 Exercise

Answer 1

$$\begin{array}{rcll} x^2 + 5y^2 & = & 81 & \\ x^2 + y^2 & = & 17 & \quad \text{SUBTRACT} \\ \hline 4y^2 & = & 64 & \\ y^2 & = & 16 & \\ y & = & \pm 4 & \end{array}$$

From the graph, four points are expected, so at this stage they are shaping up to be,

$$(\quad, -4), \ (\quad, -4), \ (\quad, 4) \text{ and } (\quad, 4)$$

The *x* part of these coordinates are easiest to find from the equation of the circle,

$$\begin{aligned} x^2 + y^2 &= 17 \\ x^2 &= 17 - y^2 \\ &= 17 - (\pm 4)^2 \\ &= 17 - 16 \\ &= 1 \\ \therefore \ x &= \pm 1 \end{aligned}$$

The points are,

$$(-1, -4), \ (1, -4), \ (-1, 4) \text{ and } (1, 4)$$

Answer 2

$$\begin{aligned} x^2 + 12y^2 &= 88 \\ x^2 + y^2 &= 66 \qquad \text{SUBTRACT} \\ \hline 11y^2 &= 22 \\ y^2 &= 2 \\ y &= \pm\sqrt{2} \end{aligned}$$

From the graph, four points are expected, so at this stage they are shaping up to be,

$(\quad , -\sqrt{2})$, $(\quad , -\sqrt{2})$, $(\quad , \sqrt{2})$ and $(\quad , \sqrt{2})$

The x part of these coordinates are easiest to find from the equation of the circle,

$$\begin{aligned} x^2 + y^2 &= 66 \\ x^2 &= 66 - y^2 \\ &= 66 - (\pm\sqrt{2})^2 \\ &= 66 - 2 \\ &= 64 \\ \therefore\ x &= \pm 8 \end{aligned}$$

The points are,

$(-8, -\sqrt{2})$, $(8, -\sqrt{2})$, $(-8, \sqrt{2})$ and $(8, \sqrt{2})$

Answer 3

Square both sides of the equation of the hyperbola,

$$y = \frac{12}{x} \Rightarrow y^2 = \frac{144}{x}$$

Then substitute this in place of the y^2 in the equation for the ellipse,

$$x^2 + 4\left(\frac{144}{x^2}\right) = 52$$

Multiply through by x^2

$$x^2 \times x^2 + \frac{x^2}{1} \times \left(\frac{576}{x^2}\right) = x^2 \times 52$$

$$x^4 + 576 = 52x^2$$

$$x^4 - 52x^2 + 576 = 0$$

$$\text{Let } z = x^2 \Rightarrow z^2 = x^4$$

$$z^2 - 52z + 576 = 0$$

$$(z - 16)(z - 36) = 0 \qquad \text{with help from list provided}$$

$$\text{Either} \quad z - 16 = 0 \quad \text{or} \quad z - 36 = 0$$

$$z = 16 \quad \text{or} \quad z = 36$$

$$\therefore \; x^2 = 16 \text{ or } x^2 = 36$$

$$x = \pm 4 \quad \text{or} \quad x = \pm 6$$

Use the equation for the hyperbola to get the y part of the points,

$$y = \frac{12}{x}$$

The four points of intersection are,

$$(-4, -3), (4, 3), (-6, -2) \text{ and } (6, 2)$$

Answer 4

Rearrange the equation of the quadratic,

$$y = \frac{x^2}{8} - 6$$

$$y + 6 = \frac{x^2}{8}$$

$$x^2 = 8y + 48$$

Then substitute this in place of the x^2 in the equation for the ellipse,

$$\left(x^2 \right) + 2y^2 = 72$$

$$\left(8y + 48 \right) + 2y^2 = 72$$

$$2y^2 + 8y - 24 = 0$$

$$y^2 + 4y - 12 = 0$$

$$(y + 6)(y - 2) = 0$$

Either $y + 6 = 0$ or $y - 2 = 0$

$$y = -6 \quad \text{or} \quad y = 2$$

Back with the equation for the quadratic,

$$x^2 = 8y + 48$$

square root both sides

$$x = \pm\sqrt{8y + 48}$$

So when $y = -6$, $x = \pm\sqrt{8 \times (-6) + 48} \Rightarrow x = 0$

And when $y = 2$, $x = \pm\sqrt{8 \times 2 + 48} \Rightarrow x = \pm 8$

The three points of intersection are,

$$(0, -6), (-8, 2) \text{ and } (8,2)$$