5.4 Answers

GCSE and Preparatory A-Level Mathematics
Conic Sections (Simultaneous Equations IV)

5.4.1 Solutions to Teaching Video "Q Formula In Use"

$$
\begin{aligned}
x^2 + 3y^2 &= 100 \\
x^2 + 3\left[\,(x+4)^2\,\right] &= 100 \\
x^2 + 3\left[\,(x+4)\,(x+4)\,\right] &= 100 \\
x^2 + 3\left[x^2 + 8x + 16\right] &= 100 \\
x^2 + 3x^2 + 24x + 48 &= 100 \\
4x^2 + 24x - 52 &= 0 \\
x^2 + 6x - 13 &= 0 \\
x &= \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times (-13)}}{2 \times 1} \\
&= \frac{-6 \pm \sqrt{36 + 52}}{2} \\
&= \frac{-6 \pm \sqrt{88}}{2} \\
&= \frac{-6 \pm \sqrt{4 \times 22}}{2} \\
&= \frac{-6 \pm 2\sqrt{22}}{2} \\
&= \frac{-6}{2} \pm \frac{2\sqrt{22}}{2} \\
&= -3 \pm \sqrt{22}
\end{aligned}
$$

Using $y = x + 4$ to get the y part of the coordinates

$\left(-3 - \sqrt{22},\ 1 - \sqrt{22}\right)$ $\left(-3 + \sqrt{22},\ 1 + \sqrt{22}\right)$

5.4.2 Solutions to 5.3 Exercise

Answer 1

$$\left(-3-\sqrt{22},\ 1-\sqrt{22}\right) \quad \left(-3+\sqrt{22},\ 1+\sqrt{22}\right)$$

These become, to one decimal place,

$$(-7.7, -3.7) \quad (1.7,\ 5.7)$$

which do indeed match up with the graph.

Answer 2

(i)

$$\frac{9 \pm \sqrt{18}}{3} = \frac{9 \pm \sqrt{9 \times 2}}{3}$$
$$= 3 \pm \sqrt{2}$$

(ii)

$$\frac{35 \pm \sqrt{50}}{5} = \frac{35 \pm \sqrt{25 \times 2}}{5}$$
$$= 7 \pm \sqrt{2}$$

(iii)

$$\frac{56 \pm \sqrt{48}}{4} = \frac{56 \pm \sqrt{16 \times 3}}{4}$$
$$= 14 \pm \sqrt{3}$$

(iv)

$$\frac{-24 \pm \sqrt{640}}{8} = \frac{-24 \pm \sqrt{64 \times 10}}{8}$$
$$= -3 \pm \sqrt{10}$$

Answer 3

(i)

$$\frac{10 \pm \sqrt{300}}{5} = \frac{10 \pm \sqrt{100 \times 3}}{5}$$
$$= 2 \pm 2\sqrt{3}$$

(ii)

$$\frac{-45 \pm \sqrt{360}}{3} = \frac{-45 \pm \sqrt{36 \times 10}}{3}$$
$$= -15 \pm 2\sqrt{10}$$

Answer 4

(i)

$$x^2 + 8x + 5 = 0$$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$= \frac{-8 \pm \sqrt{64 - 20}}{2}$$

$$= \frac{-8 \pm \sqrt{44}}{2}$$

$$= \frac{-8 \pm \sqrt{4 \times 11}}{2}$$

$$= -4 \pm \sqrt{11}$$

(ii)

$$x^2 + 2x - 9 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-9)}}{2 \times 1}$$

$$= \frac{-2 \pm \sqrt{4 + 36}}{2}$$

$$= \frac{-2 \pm \sqrt{40}}{2}$$

$$= \frac{-2 \pm \sqrt{4 \times 10}}{2}$$

$$= -1 \pm \sqrt{10}$$

Answer 5

$$x^2 + 6y^2 = 84$$
$$x^2 + 6\left[\,(x+7)^2\,\right] = 84$$
$$x^2 + 6\left[\,(x+7)(x+7)\,\right] = 84$$
$$x^2 + 6\left[x^2 + 14x + 49\right] = 84$$
$$x^2 + 6x^2 + 84x + 294 = 84$$
$$7x^2 + 84x + 210 = 0$$
$$x^2 + 12x + 30 = 0$$
$$x = \frac{-12 \pm \sqrt{12^2 - 4 \times 1 \times 30}}{2 \times 1}$$
$$= \frac{-12 \pm \sqrt{144 - 120}}{2}$$
$$= \frac{-12 \pm \sqrt{24}}{2}$$
$$= \frac{-12 \pm \sqrt{4 \times 6}}{2}$$
$$= \frac{-12 \pm 2\sqrt{6}}{2}$$
$$= \frac{-12}{2} \pm \frac{2\sqrt{6}}{2}$$
$$= -6 \pm \sqrt{6}$$

Using $y = x + 7$ to get the y part of the coordinates

$$\left(-6 - \sqrt{6},\ 1 - \sqrt{6}\right) \quad \left(-6 + \sqrt{6},\ 1 + \sqrt{6}\right)$$