6.4 Answers

GCSE and Preparatory A-Level Mathematics

Conic Sections (Simultaneous Equations IV)

Answer 1

$$2x^2 + 8x + 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 2$, $b = 8$ and $c = 5$ gives

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 5}}{2 \times 2}$$

$$= \frac{-8 \pm \sqrt{64 - 40}}{4}$$

$$= \frac{-8 \pm \sqrt{24}}{4}$$

$$= \frac{-8}{4} \pm \frac{\sqrt{4 \times 6}}{4}$$

$$= -2 \pm \frac{2\sqrt{6}}{4}$$

$$= -2 \pm \frac{\sqrt{6}}{2}$$

Answer 2

$$3x^2 + 2x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 3$, $b = 2$ and $c = -4$ gives

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times (-4)}}{2 \times 3}$$

$$= \frac{-2 \pm \sqrt{4 + 48}}{6}$$

$$= \frac{-2 \pm \sqrt{52}}{6}$$

$$= \frac{-2}{6} \pm \frac{\sqrt{4 \times 13}}{6}$$

$$= -\frac{1}{3} \pm \frac{2\sqrt{13}}{6}$$

$$= -\frac{1}{3} \pm \frac{\sqrt{13}}{3}$$

Answer 3

$$3x^2 + 6x - 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 3$, $b = 6$ and $c = -5$ gives

$$x = \frac{-6 \pm \sqrt{6^2 - 4 \times 3 \times (-5)}}{2 \times 3}$$

$$= \frac{-6 \pm \sqrt{36 + 60}}{6}$$

$$= \frac{-6 \pm \sqrt{96}}{6}$$

$$= \frac{-6}{6} \pm \frac{\sqrt{16 \times 6}}{6}$$

$$= -1 \pm \frac{4\sqrt{6}}{6}$$

$$= -1 \pm \frac{2\sqrt{6}}{3}$$

To 3 significant figures the solutions are -2.63 or 0.633

Answer 4

$$x^2 + 90x - 1200 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 1$, $b = 90$ and $c = -1200$ gives

$$x = \frac{-90 \pm \sqrt{90^2 - 4 \times 1 \times (-1200)}}{2 \times 1}$$

$$= \frac{-90 \pm \sqrt{8100 + 4800}}{2}$$

$$= \frac{-90 \pm \sqrt{12900}}{2}$$

$$= \frac{-90}{2} \pm \frac{\sqrt{100 \times 129}}{2}$$

$$= -45 \pm \frac{10\sqrt{129}}{2}$$

$$= -45 \pm 5\sqrt{129}$$

To 3 significant figures the solutions are -102 or 11.8

Answer 5

$$3x^2 + 8x + 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 3,\ b = 8$ and $c = 1$ gives

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 3 \times 1}}{2 \times 3}$$

$$= \frac{-8 \pm \sqrt{64 - 12}}{6}$$

$$= \frac{-8 \pm \sqrt{52}}{6}$$

$$= \frac{-8}{6} \pm \frac{\sqrt{4 \times 13}}{2}$$

$$= -\frac{4}{3} \pm \frac{2\sqrt{13}}{6}$$

$$= -\frac{4}{3} \pm \frac{\sqrt{13}}{3}$$

To 2 decimal places the solutions are -0.13 or -2.54

Answer 6

$$2x^2 + 3y^2 = 11$$

$$2(3y - 1)^2 + 3y^2 = 11$$

$$2[(3y - 1)(3y - 1)] + 3y^2 = 11$$

$$2[9y^2 - 6y + 1] + 3y^2 = 11$$

$$21y^2 - 12y - 9 = 0$$

$$7y^2 - 4y - 3 = 0$$

(could use formula here)

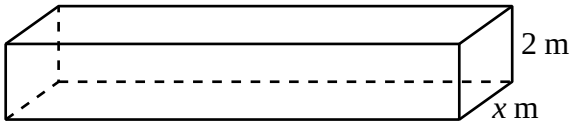
$$(7y + 3)(y - 1) = 0$$

Either $y = -\frac{3}{7}$ or $y = 1$

Using $x = 3y - 1$ to get the x part of the coordinates

$$\left(-\frac{16}{7}, -\frac{3}{7}\right) \quad (2,\ 1)$$

Answer 7



(a)

$$Volume = length \times breadth \times height$$

$$45 = 2x(x + 7)$$

$$45 = 2x^2 + 14x$$

$$2x^2 + 14x - 45 = 0 \qquad \square$$

(b)

$$2x^2 + 14x - 45 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 2$, $b = 14$ and $c = -45$ gives

$$x = \frac{-14 \pm \sqrt{14^2 - 4 \times 2 \times (-45)}}{2 \times 2}$$

$$= \frac{-14 \pm \sqrt{196 + 360}}{4}$$

$$= \frac{-14 \pm \sqrt{556}}{4}$$

$$= \frac{-14}{4} \pm \frac{\sqrt{4 \times 139}}{4}$$

$$= -\frac{7}{2} \pm \frac{\sqrt{139}}{2}$$

To 1 decimal places the solutions are -9.4 or 2.4

Breadth is 2.4 metres (as it cannot be negative)

Answer 8

(a)

Area of a trapezium is half the sum of the parallel sides time the distance in between

$$\therefore \quad 17 = \frac{1}{2}(x + (x + 7)) \times 2x$$

$$2 \times 17 = (2x + 7) \times 2x$$

$$34 = 4x^2 + 14x$$

$$4x^2 + 14x - 34 = 0$$

$$2x^2 + 7x - 17 = 0 \qquad \square$$

(b)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

with $a = 2,\ b = 7$ and $c = -17$ gives

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \times 2 \times (-17)}}{2 \times 2}$$

$$= \frac{-7 \pm \sqrt{49 + 136}}{4}$$

$$= \frac{-7 \pm \sqrt{185}}{4}$$

To 3 significant figures the solutions are -5.15 or 1.65

In the context of the problem only $x = 1.65$ is a valid solution.